Tests of Backscatter Coefficient Measurement Using Broadband Pulses

Jian-Feng Chen, James A. Zagzebski, *Senior Member, IEEE*, and Ernest L. Madsen

***Abstract*—An adaptation to a data reduction method is outlined for determining backscatter coefficients, η when broad bandwidth pulses are employed. The accuracy of these η values is assessed with well-characterized phantoms, which have independently calculated backscatter coefficients based on their physical properties. One phantom produces Rayleigh-like scattering, where the backscatter coefficient varies smoothly with frequency over the analysis bandwidth. A second phantom exhibits local maxima and minima in the scattering function versus frequency due to presence of millimeter sized graphite gel spheres in a gel background. The method was found to produce accurate results using time gate durations as small as 2 μs, although better accuracy is obtained for longer gate durations, particularly when the sample exhibits resonance peaks in backscatter versus frequency. Use of a Hamming window in place of a rectangular window extends the accuracy near the upper and lower limits of the frequency range.**

I. Introduction

In medical ultrasound, the backscatter coefficient is commonly used to quantify the scattering properties of biological tissue [1]–[3]. The backscatter coefficient is defined as the differential scattering cross section per unit volume for a scattering angle of 180°. Measurements of this quantity involve the projection of a pulsed ultrasound beam into a volume containing the medium of interest and monitoring echo signals due to scattering. Data reduction methods are then applied to relate the echo signal spectral density to the backscatter coefficient, accounting for instrumental and transmission path dependencies on the amplitude of these signals.

A data reduction method that yields accurate backscatter coefficients from pulse echo data has been described previously by the authors [4] and its accuracy verified [5]. The accuracy of the method is related to the fact that detailed pressure fields are included, enabling measurements to be obtained anywhere in the field of the transducer, and the temporal nature of the measurement process is maintained throughout the data reduction. This method is most frequently applied for insonification by long duration acoustic pulses and for narrow bandwidth conditions.

Often it is of interest to obtain backscatter coefficients results quickly over a range of frequencies. In our lab this has been the case in *in vivo* measurements and in characterizing short-lived ultrasound contrast agents, and is done using insonification pulses that have a broad frequency spectrum. When broadband pulses are employed, an important consideration of the date reduction method is that the frequency dependence of scattering generally must be included in the data analysis [5]. Madsen [6] describes an iterative technique for carrying this out when short gates are involved. Alternatively, Wear *et al.* [7] have outlined broadband adaptations of the original data reduction technique described by Madsen. Boote *et al.* [8] reported accurate results for broadband pulses when the analysis frequency is restricted to the central frequency of the echo pulse and scattering varies smoothly with frequency.

The purpose of the work described in this paper is to evaluate the accuracy of backscatter coefficient measurements under broadband insonification conditions when the analysis is extended to the entire frequency spectrum of the pulse. Similar work using phantoms was done by Wear *et al;* however, the absolute accuracy of backscatter coefficients was not considered by them. The present work was performed using well-characterized phantom materials, including one which exhibits resonance-like peaks in the backscatter over the pulse spectrum. In the present paper, both rectangular and Hamming windows of various durations were used in the echo data reduction to evaluate effects of gating functions on the results.

II. Broadband Analysis Method

In measuring the backscatter coefficient, the beam from a pulsed transducer is directed into a volume containing scatterers. Echo signals are detected by the same transducer and stored for off-line analysis. Assumptions made in the data reduction are, first, the scattered wave fronts are assumed to be spherical in the region of the transducer face. This requires that the scattering sources either be monopolar in nature or be sufficiently far from the transducer face that this assumption is valid. Second, the scatterers are assumed to be discrete and spatially randomly distributed. Thus the echo signal is a superposition of waves from all scatterers in the transducer field. Scatterers are described in terms of an angular distribution function [9] $\phi(\omega)$, evaluated at 180°.

The backscatter coefficient as a function of frequency $\eta(\omega)$, is given by (1) (see top of next page) [4] where the numerator is the measured echo signal spectral density function, $T(\omega)B(\omega)$ accounts for the system frequency response, and $A(\mathbf{r},\omega)$ represents the transducer field. $W(\omega - \omega')$ is the Fourier transform of a time gate used to select echo signals from the region of interest, and $\langle N\rangle$ is the scatterer number density. The term $g(\omega')$ represents the frequency dependence of the scattering function, with $g(\omega)$ the value at the analysis

Manuscript received October 5, 1992; revised March 12, 1993; accepted March 22, 1993. This work was supported in part by the National Institutes of Health under Grant R01-CA39224.

The authors are with the Department of Medical Physics, University of Wisconsin, Madison, WI 53706.

IEEE Log Number 9210380.

0885–3010/93$03.00 © 1993 IEEE

$$\begin{aligned}\boldsymbol{\eta}(\boldsymbol{\omega}) &= \langle N\rangle ||\phi(\boldsymbol{\omega})||^2 \\ &\simeq \frac{\langle V_g(\boldsymbol{\omega})V_g^*(\boldsymbol{\omega})\rangle}{\iiint\limits_{\Omega} d\boldsymbol{r}\left\| \int_{-\infty}^{+\infty} d\boldsymbol{\omega}' T(\boldsymbol{\omega}')B_0(\boldsymbol{\omega}')\frac{g(\boldsymbol{\omega}')}{g(\boldsymbol{\omega})}W(\boldsymbol{\omega}-\boldsymbol{\omega}')[A(\boldsymbol{r},\boldsymbol{\omega}')]^2\right\|^2}\end{aligned} \tag{1}$$

frequency. The volume integral Ω, includes all scatterers which can contribute to the time-gated signal.

In the present paper, the transducer is excited with a broadband pulse and a time-gate whose frequency bandwidth is much smaller than the frequency range of the echo signals is used. This allows us to ignore the effects of the convolution with the Fourier transform of the gate function in (1). Mathematically, this approximation is

$$W(\boldsymbol{\omega}-\boldsymbol{\omega}') \simeq C \cdot \boldsymbol{\delta}(\boldsymbol{\omega}-\boldsymbol{\omega}')$$

where C is a constant that depends on the gating function. (i.e., $C = 1$ for a rectangular window; $C = 0.63$ for a Hamming window). With the above simplification, the expression for the frequency dependent backscatter coefficient is given by

$$\boldsymbol{\eta}(\boldsymbol{\omega}) \approx \frac{\langle V_g(\boldsymbol{\omega})V_g^*(\boldsymbol{\omega})\rangle}{C^2||T(\boldsymbol{\omega})B_0(\boldsymbol{\omega})||^2 \iiint\limits_{\Omega} d\boldsymbol{r}||A(\boldsymbol{r},\boldsymbol{\omega})||^4} \tag{2}$$

where the ratio $g(\omega')/g(\omega)$ is taken as 1. This equation is similar to that derived by Wear *et al.* [7], who also based their analysis on the Madsen *et al.* method [4]. However, here no approximation is made regarding the range dependence of $A(\boldsymbol{r},\omega)$, resulting in no compromise as the gate duration is increased.

The echo from a planar reflector submerged in water and oriented with its reflecting surface perpendicular to the incident ultrasound beam is used to determine the product $T(\omega)B_0(\omega)$ [4]. $A(\boldsymbol{r},\omega)$ is computed using the diameter and focal properties of the transducer [10].

III. Experimental Procedure

Conditions for which (2) yields valid results were evaluated using experimental apparatus described in a previous paper [11]. In the present paper, a 13.6-mm diameter, 2.5-MHz center frequency transducer, having a 67% -6-dB frequency bandwidth (Echo Ultrasound, Reedsville, PA) was used to acquire echo data. The transducer diameter was verified by searching for the last axial minima in water using a 0.6-mm diameter hydrophone when the transducer was driven with a 2.5-MHz tone burst.

Backscatter results were obtained using two different phantoms. One consisted of graphite-laced agar having randomly positioned glass bean scatterers. The glass bead scatterers have a mean diameter of 89 μm, with a standard deviation of 7 μm. The speed of sound and ultrasonic attenuation coefficient are 1580 m/s and 1.23 dB/cm at 2.5 MHz. The second phantom consists of 0.57 ± 0.05 mm diameter graphite-laced agar spheres randomly distributed in a background of plain agar. The speed of sound and ultrasonic attenuation coefficient are 1560 m/s and 0.28 dB/cm at 2.5 MHz.

For both phantoms, the expected backscatter coefficients versus frequency were calculated using the theory of Faran [12]. Parameters used in these calculations are the scatterer mass density (glass 2.38 g/cm^3, agar 0.94 g/cm^3, and agar spheres 1.034 g/cm^3), Poisson's ratio (glass 0.21 and agar 0.4993), the longitudinal speed of sound (glass 5770 m/s, agar 1540 m/s, and water 1483 m/s), and the diameter distributions of the scatterers.

Echo signals from the test phantoms were recorded at 149 statistically independent positions by lateral translations of the phantom between recordings. Discrete Fourier transforms of each of these 149 waveforms were then computed yielding the values of $V_g(\omega)$. The mean value of the square moduli of the Fourier transform then yields $\langle V_g(\omega)V_g^*(\omega)\rangle$. The volume integration in (2) was done using 100 lateral points, extending 3 cm from the beam axis, and 10 axial points evenly distributed over the depth interval $c_p(t_0 - \tau_{pd})/2$ to $c_p(t_0 - \tau)/2$ where c_p is the speed of sound in the test phantom, t_0 is the onset time of the gate, τ is the time gate duration and τ_{pd} is the pulse duration.

IV. Results and Discussion

Frequency dependent backscatter coefficients obtained for different gate durations and for both a rectangular and a Hamming window, are shown in Fig. 1 for the glass bead scatterer phantom. The theoretical results computed using the Faran equations [12] show that the backscatter coefficient varies smoothly with frequency from 1 to 4 MHz. The experimental results for each type of gate function are in agreement with the theoretical results near the 2.5-MHz center frequency of the pulse. As expected, however, the degree of the agreement breaks down for analysis frequencies near the edge of the spectrum, particularly for the shortest gate duration (Fig. 1(c)). The beat overall results were with the 20-μs Hamming window (Fig. 1(a)).

For the 5- (Fig. 1(b)) and 20-μs time gates a noticeable improvement in the result near the edge of the spectrum is seen with the Hamming window compared to the rectangular window. Evidently, the reduced sidelobe leakage for the Hamming gate [13] has it greatest impact on these measurements for frequencies near the edge of the pulse spectrum.

Prior to our work, Wear *et al.* [7] compared backscatter results for different gate durations for a phantom containing glass beads, though these investigations did not verify absolute scatter levels. Their results for gate durations of 3.2 and

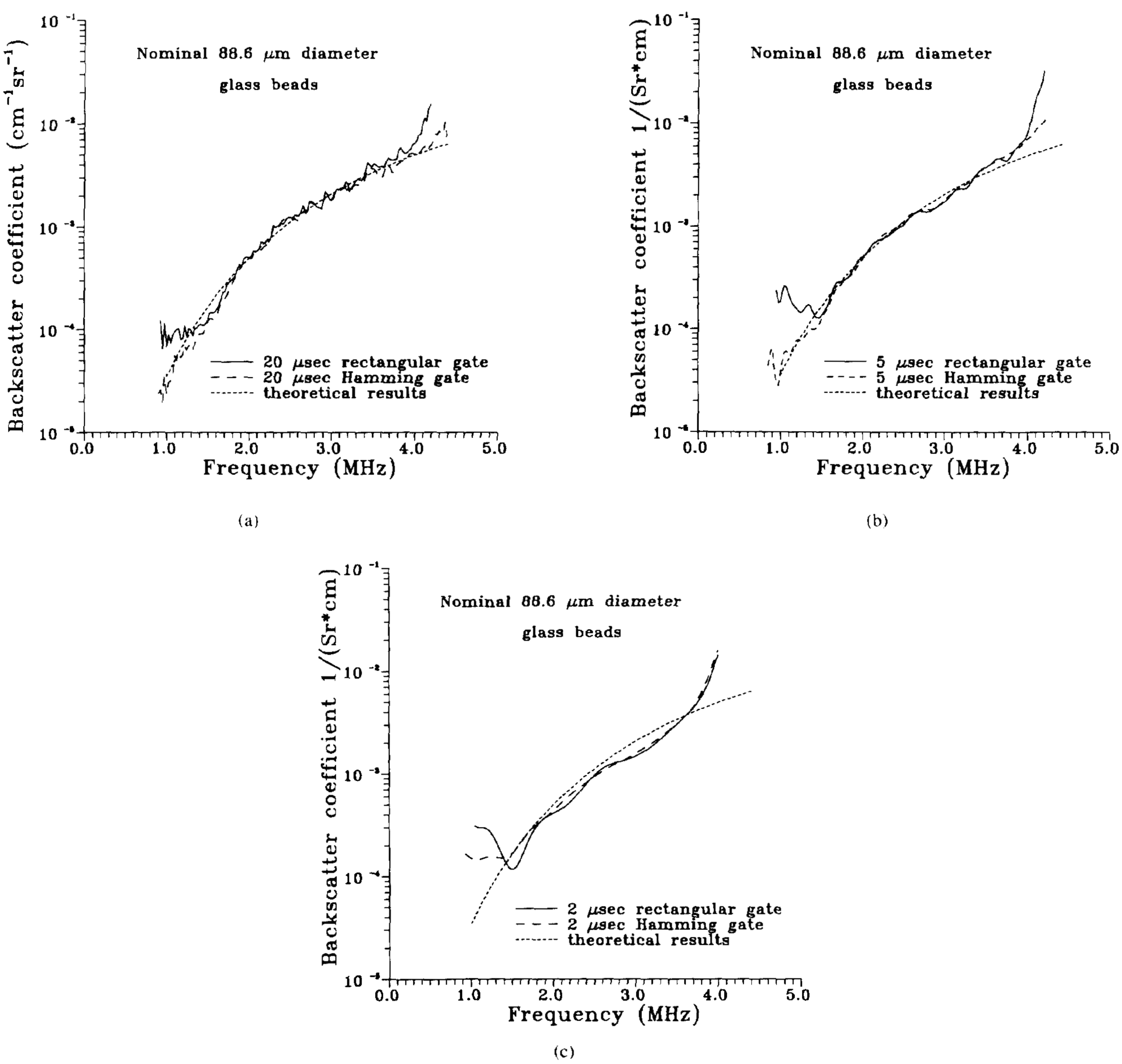
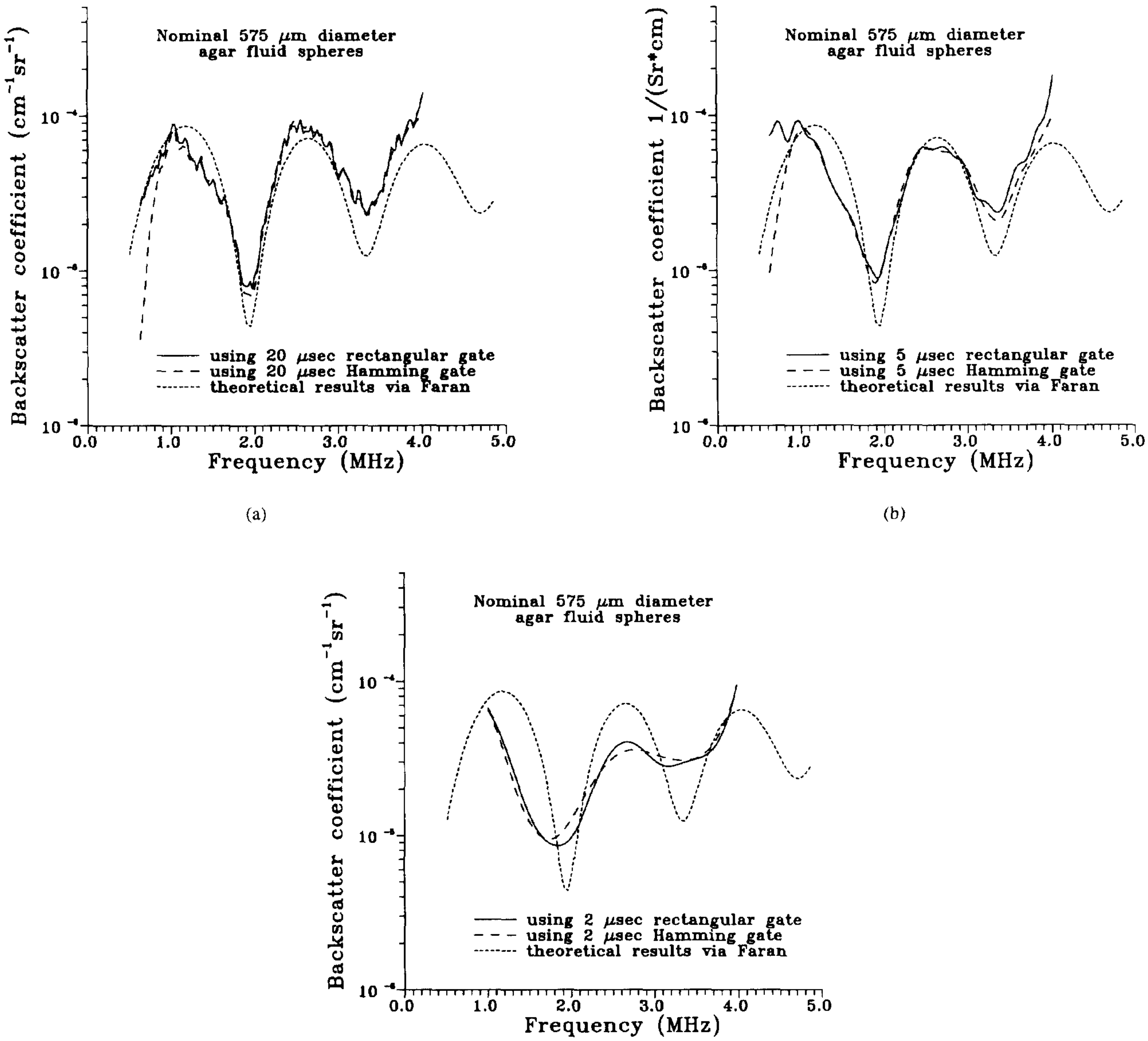


Fig. 1. The experimental values of the frequency dependent backscatter coefficient for the glass bead scatterer phantom, using 20-, 5-, and 2-μs gate durations. Also shown are the predicted values based on Faran's theory.

6.4 μs were identical, and were noticeably different from results obtained both with 1.6- and 12.8-μs gates, leading to the conclusion that the 3.2- and 6.4-μs gates yielded the more accurate results. However, our results for the longest gate duration (20 μs) agree with the expected scattering over the greatest extent of the frequency spectrum. In contrast to Wear's approach, in our analysis, individual calculations of the transducer field, $A(\boldsymbol{r}, \omega)$ are done throughout the gated depth region, probably accounting for difference between their results and ours.

Results for the agar sphere phantom are presented in Fig. 2, where the theoretical predictions indicate that the backscatter coefficient has several peaks between 1 and 4 MHz. This phantom was designed to provide stronger challenges to data reduction algorithms, where the analysis involves more complicated frequency dependencies. Fig. 2(a) and (b) shows the experimental results of the backscatter coefficient for relatively long gate durations (gate duration $\tau = 20$ and $5\,\mu$s). Fig. 2(c) shows the backscatter coefficients for a relatively short gate duration (gate duration $\tau = 2\,\mu$s). For longer gate durations, the experimental results again suggest that the Hamming time gate function actually is slightly better than the rectangular time gate function, as indicated by results slightly closer to the expected behavior at 1.8 MHz. For 2-μs gates the inability of the experimental results to track the expected behavior is clear for both gate types. Interestingly, for the shorter gate durations, the results using the rectangular window are actually slightly better than the results using the Hamming window, as indicated near the peaks and valleys of the backscatter spectra. The rectangular gate results follow the predicted minimum and maximum slightly better. Evidently the width of the main lobe is relatively important compared with the side lobe leakage



(a) (b) (c)

Fig. 2. The experimental values of the frequency dependent backscatter coefficient for the millimeter sized gel-sphere phantom, using 20-, 5-, and 2-μs gate durations. Also shown are the predicted values based on Faran's theory.

for tracking this more complex scattering behavior, and a rectangular window has a smaller main lobe width than the Hamming window. It should be pointed out that part of the discrepancy between theory and experiment may be related to uncertainties in the properties of the phantom. The spheres were sieved to yield a tight diameter distribution, but the magnitude of the errors in the theoretical calculations for the phantom are not known.

Another important test of a backscatter measurement method is whether results are independent of transducer-to-sample distance [5]. Fig. 3 shows backscatter coefficients versus frequency for sample depths (center of the time gate) of 5, 10, 12 and 15 cm. A 10-μs Hamming gate is used in the analysis. Results for 1.5–3.2 MHz are hardly distinguished for the various depths; however, at 3.5 MHz and higher, the 5-cm results are more than 50% greater than the expected scattering. Thus results with the sample close to the transducer exhibit the greatest errors, particularly near the edges of the pulse spectrum. The results demonstrate that the frequency range for accurate results may be more limited in the near field of the transducer.

V. Conclusion

The method for determining backscatter coefficients, as described and implemented in this paper, appears to work well for determining the frequency dependent backscatter coefficients in volumes where these parameters can be considered uniform, especially when the frequency dependence of the backscatter coefficient follows a simple power law over the echo signal spectrum. The difference between theoretical and experimental results is minimized using a relatively long gate duration applied to the echo signals, as noted previously [5].

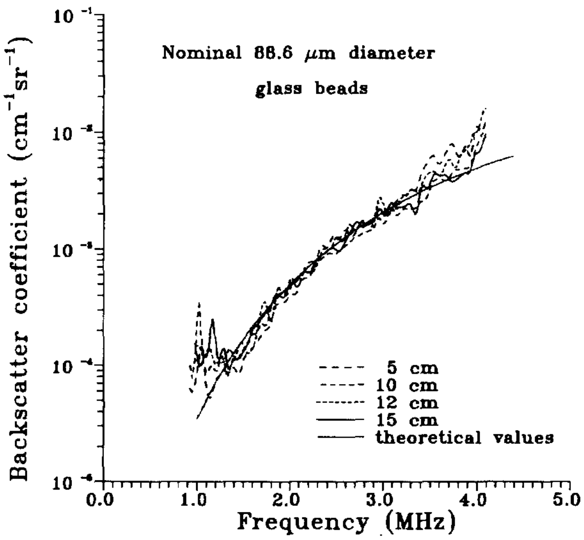


Fig. 3. The experimental values of the frequency dependent backscatter coefficient for the glass bead scatterer phantom, using a 10-μs Hamming window and at different sample depths. Also shown are the predicted values based on Faran's theory.

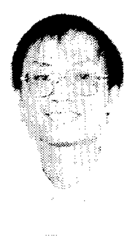
In the absence of resonance peaks in the frequency dependent backscatter coefficient, the results using a rectangular window generally will be worse than the results using other window functions with lower sidelobes. On the other hand, if the peaks of the frequency dependent backscatter coefficient are close to each other, as in the agar sphere phantom described here, the results using a rectangular window could be more accurate than results using other window functions, for example, a Hamming window, because of the latter's wider main-lobe. The Hamming window extends the accuracy of backscatter coefficient determinations near the upper and lower limits of the frequency range. This paper points out the importance of verifying backscatter data reduction methodology prior to use in clinical determinations.

Acknowledgment

The authors are grateful to Gary Frank and Michael Macdonald for their assistance in constructing the agar-sphere phantom and to Clyde Oakley of Echo Ultrasound for use of the broadband transducer.

References

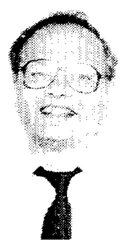
[1] R. A. Sigelmann and J. M. Reid, "Analysis and measurement of ultrasound backscattering from an ensemble of scatterers excited by sine-wave bursts," *J. Acoust. Soc. Amer.*, vol. 53, pp. 1351–1355, 1973.
[2] R. C. Chivers and C. R. Hill, "A spectral approach to ultrasonic scattering from human tissues: Methods, objectives, and backscatter measurement," *Phys. Med. Biol.*, vol. 20, pp. 799–815, 1975.
[3] D. Nicholas, "Orientation and frequency dependence of backscattered energy and its clinical application," in *Recent Advances in Ultrasound and Medicine*, D. N. White, ed. New York: Research Studies, 1977, pp. 29–54.
[4] E. L. Madsen, M. F. Insana, and J. A. Zagzebski, "Method of data reduction for accurate determination of acoustic backscatter coefficients," *J. Acoust. Soc. Amer.*, vol. 76, pp. 913–923, 1984.
[5] T. J. Hall, "Experimental methods for accurate determination of acoustic backscatter coefficient," Ph.D. dissertation, Univ. of Wisconsin, Madison, 1988.
[6] E. L. Madsen, "Method of determination of acoustic backscatter and attenuation coefficient independent of depth and instrumentation," in *Ultrasonic Scattering in Biological Tissues*, K. K. Shung and G. A. Thieme, eds. Boca Raton, FL: CRC, 1992.
[7] K. A. Wear, M. R. Milunski, S. A. Wickline, J. E. Perez, B. E. Sobel, and J. G. Miller, "Differentiation between acutely ischemic myocardium and zones of completed infarction in dogs on the basis of frequency-dependent backscatter," *J. Acoust. Soc. Amer.*, vol. 85, pp. 2634–2641, 1989.
[8] E. J. Boote, T. J. Hall, E. L. Madsen, and J. A. Zagzebski, "Improved resolution backscatter coefficient imaging," *Ultrasonic Imaging*, vol. 13, pp. 437–459, 1991.
[9] P. M. Morse, and K. U. Ingard, *Theoretical Acoustics.* New York: McGraw-Hill, 1969, ch. 8.
[10] E. L. Madsen, M. M. Goodsitt, and J. A. Zagzebski, "Continuous wave generated by focused radiators," *J. Acoust. Soc. Amer.*, vol. 70, pp. 1508–1517, 1981.
[11] M. F. Insana, E. L. Madsen, T. J. Hall, and J. A. Zagzebski, "Tests of the accuracy of a data reduction method for determination of acoustic backscatter coefficients," *J. Acoust. Soc. Amer.*, vol. 79, pp. 1230–1236, 1986.
[12] J. J. Faran, Jr., "Sound scattering by solid cylinders and spheres," *J. Acoust. Soc. Amer.*, vol. 23, pp. 405–418, 1951.
[13] A. V. Oppenheim and R. W. Schafer, *Digital Signal Processing.* Englewood Cliffs, NJ: Prentice-Hall, 1975.
[14] P. N. T. Wells, *Biomedical Ultrasonics.* New York: Academic, 1977, p. 118.



Jian-Feng Chen was born in Hangzhou, China. He received the B.S. degree in physics from Tongji University, Shanghai, China in 1985, and the M.S. degrees in applied physics, nuclear engineering, and medical physics from Tongji University, University of Missouri and University of Wisconsin-Madison, in 1988, 1991, and 1992, respectively. Now he is working toward a Ph.D. in medical physics.

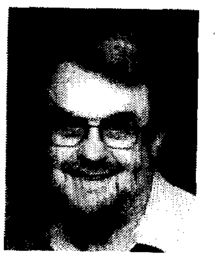
His research interests are in the medical ultrasound and magnetic resonance imaging and tissue characterization.

Mr. Chen is a student member of the American Association of Physicists in Medicine and the Acoustical Society of America.



James A. Zagzebski (S'83–M'85–SM'91) received the B.S. degree in physics from St. Mary's College, Winona, MN, and the M.S. degree in physics and the Ph.D. degree in radiological sciences from the University of Wisconsin, Madison.

He is a Professor of Medical Physics and of Radiology and Human Oncology at the University of Wisconsin. His research interests include medical imaging, ultrasound methods for imaging and tissue characterization, flow visualization using ultrasound, and methods of accessing performance of ultrasound scanners.



Ernest L. Madsen received the B.S. and the M.S. degrees in physics from the University of Maryland, College Park, and the Ph.D. degree in physics from the Catholic University of America, Washington, DC.

He is an Associate Professor of Medical Physics of the University of Wisconsin, Madison. His research encompasses a range of subjects in medical ultrasound, hyperthermia, and biological effects of ultrasound. He also pursues development of test objects for use in magnetic resonance imaging and magnetic resonance spectroscopy.