Non-Gaussian Versus Non-Rayleigh Statistical Properties of Ultrasound Echo Signals

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***Abstract*— Parameters expressing the non-Gaussian and the non-Rayleigh properties of ultrasound echo signals are derived for the case of a pulsed transducer insonifying a medium containing randomly distributed scatterers. Both parameters depend on the measurement system, including the transducer field and pulse frequency content, as well as on the medium's properties. The latter is expressed in terms of the number of scatterers per unit volume and the second and fourth moments of the medium's scattering functions. A simple relationship between the parameters describing the non-Gaussian and non-Rayleigh properties is derived and verified experimentally.**

I. INTRODUCTION

SEVERAL articles have been published in recent years that describe the use of high moments of ultrasonic echo signals to estimate the scatterer concentration in an insonified medium. Earlier literature described applications of these techniques in underwater acoustics [1], [2], [3] while more recent papers [4], [5] involve their use in medical ultrasonography. In each case, the experimenters compute the ratio of the fourth moment to the square of the second moment of the time-domain echo signal. However, substantial differences exist among the expressions for the scatterer number density computed by the various authors and in the interpretation of the results. In particular, Sleefe and Lele [4] apply their analysis to the RF echo signal and relate the scatterer number density to the deviation of the above ratio from that expected for Gaussian statistics. Weng and Reid [5] compute the ratio of the fourth to the square of the second moment of the magnitude of the echo signal. A scatterer "clustering parameter" is estimated from the deviation of this ratio from that expected if Rayleigh statistics apply.

We have also developed a method for estimating the scatterer number density [6]. In our approach, an "effective scatterer number density" is measured, which is the actual scatterer number density multiplied by a frequency dependent factor that depends on the differential scattering cross sections of all scatterers. The analysis is done in the frequency domain but it explicitly retains both the temporal nature of the data acquisition and the properties of the ultrasound field in arriving at a frequency dependent effective scatterer number density.

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The purpose of the work described in this paper is to derive relationships between non-Gaussian and/or non-Rayleigh statistical properties of echo signals and the characteristics of the insonified media. To do this, we first rederive our expressions related to the scatterer number density, but using the time-domain echo signal analogous to work by previous investigators. Equations for the scatterer number density that are similar to those of Sleefe and Lele [4] and of Weng and Reid [5] are obtained when one computes moments of the RF signal or of the envelope of the signal, respectively. The basic expressions used in our derivations involve both the properties of the medium and complete characterization of the measurement system. Thus, our results provide further insight into the meaning of the statistical parameters measured by the above authors. Under narrow bandwidth conditions, it is indicated by Denbigh and Smith [3] that a simple relationship exists for the scatterer number density computed from the moments of the radio-frequency signal and from the envelope of the signal. This relationship is derived here and is verified experimentally.

II. THEORY

A. The Non-Rayleigh Properties of the Time Domain Echo Signal Envelope

We consider a situation where a pulsed transducer is used to insonify a medium containing sparse, randomly distributed scatterers. The complex echo signal $U(t)$ can be represented as a superposition of signals due to all scatterers in the beam. That is, [7]

$$U(t) = \sum_{i=1}^{M} u_i(\boldsymbol{r}_i, t) \tag{1}$$

where

$$u_i(\boldsymbol{r}_i, t) = \int_0^{+\infty} d\omega T(\omega) A_o(\omega) e^{-i\omega t} \cdot \Psi_i(\omega) A^2(\boldsymbol{r}_i, \omega). \tag{2}$$

In this expression, $\psi_i(\omega)$ is termed "the angular distribution function" [8] at a 180° scattering angle for a scatterer at position $\boldsymbol{r}_i, T(\omega)$ is a complex transfer function relating the net instantaneous force on the transducer at the angular frequency ω to the detected voltage. $A_o(\omega)$ is a complex superposition coefficient corresponding to the frequency composition of the emitted pulse. The beam pattern of the transducer is accounted for with $A(\boldsymbol{r}, \omega)$, the Rayleigh integral for the case in which

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the normal component of the velocity at any instant of time is the same at all points on the radiating surface. It is given by [9]

$$A(\boldsymbol{r},\omega) \equiv \iint_s ds' \frac{e^{ik|\boldsymbol{r}-\boldsymbol{r}'|}}{|\boldsymbol{r}-\boldsymbol{r}'|}$$

where the integration is over the transducer surface, $\boldsymbol{r}'$ points to an element ds' on the transducer surface, and the complex wavenumber is denoted by $k \equiv \omega/c(\omega) + i\alpha(\omega)$, where $c(\omega)$ is the speed of sound and $\alpha(\omega)$ is the attenuation coefficient. The sum in (1) is over all scatterers M which significantly contribute to the echo signal at time t for each transducer location. These scatterers are located in a volume $\Delta\Omega$, which is defined by the ultrasound field produced by the transducer and by the ultrasound pulse.

When the positions of all scatterers are independent, the first moment associated with the echo signal $\langle u_i(\boldsymbol{r}_i,t)\rangle$ is approximately zero. Then, the incoherent intensity (also called the second moment) is given by

$$\begin{aligned}\langle I(t)\rangle &\equiv \langle U(t)U^*(t)\rangle \\ &= \sum_{M=0}^{+\infty} P(M)\left\langle\left\{\sum_{i=1}^{M}\sum_{j=1}^{M} u_i(\boldsymbol{r}_i,t)u_j^*(\boldsymbol{r}_j,t)\right\}\right\rangle \\ &\approx \sum_{M=0}^{+\infty} P(M)\left\{\sum_{i=1}^{M}\langle u_i(\boldsymbol{r}_i,t)u_i^*(\boldsymbol{r}_i,t)\rangle\right\}\end{aligned}$$

where $P(M)$ is the probability that there are M scatterers in $\Delta\Omega$ and the $< \cdots >$ denotes an ensemble average. The ensemble average is related to the statistical properties of the variable $u(\boldsymbol{r},t)$, i.e., it is independent of specific realizations $u_i(\boldsymbol{r}_i,t)$. Thus, the label i is unnecessary. The incoherent intensity is simplified to

$$\langle I(t)\rangle \cong \langle M\rangle\langle\|u(\boldsymbol{r},t)\|^2\rangle \tag{3}$$

where we have used the fact that

$$\langle u_i(\boldsymbol{r},t)u_j^*(\boldsymbol{r},t)\rangle = 0$$

unless $i = j$, and

$$\sum_{M=0}^{+\infty} P(M)M = \langle M\rangle. \tag{3'}$$

In (3), the quantity $\langle\|u(\boldsymbol{r},t)\|^2\rangle$ denotes the ensemble average over both scatterers and space. Let $\langle\cdots\rangle_s$ denote just the ensemble average over "sets" of scatterers, where each set includes scatterers identical to one another, on a one-to-one basis, but unlike those in other sets. Then,

$$\langle I(t)\rangle = \langle N\rangle \iiint_{\Delta\Omega} d\boldsymbol{r}\langle\|u(\boldsymbol{r},t)\|^2\rangle_s \tag{4}$$

where $\langle N\rangle$ is the scatterer number density. Equation (4) is the time domain analog to (11) in [6] which involves the frequency domain.

Similarly, because for random scatterer positions $\langle u(\boldsymbol{r},t)\rangle \approx 0$, the fourth moment of the complex echo signal is given by

$$\begin{aligned}\langle I^2(t)\rangle &\equiv \langle (U(t)U^*(t))^2\rangle \\ &= \sum_{M=0}^{+\infty} P(M) \\ &\cdot\left\{\sum_{i=1}^{M}\sum_{j=1}^{M}\sum_{k=1}^{M}\sum_{l=1}^{M}\langle u_i(\boldsymbol{r}_i,t)u_j^*(\boldsymbol{r}_j,t)u_k(\boldsymbol{r}_k,t)u_l^*(\boldsymbol{r}_l,t)\rangle\right\}.\end{aligned}$$

The contributions of terms containing $\langle u_i^m(\boldsymbol{r}_i,t)u_i^{n^*}(\boldsymbol{r}_i,t)\rangle$ are negligible when $m \neq n$, since the product varies approximately as $\exp[2i(m-n)kr_i]$. Therefore, only three classes of significant (incoherent) terms exist for each M:

1) M terms for which $i = j = k = l$,
2) $M(M-1)$ terms for which $i = j \neq k = l$,
3) $M(M-1)$ terms for which $i = l \neq k = j$.

Thus we have

$$\begin{aligned}\langle I^2(t)\rangle &\approx \sum_{M=0}^{+\infty} P(M)\left\{\sum_{i=1}^{M}\langle\|u_i(\boldsymbol{r}_i,t)\|^4\rangle\right. \\ &\quad\left. +2\sum_{i=1}^{M}\sum_{\substack{j=1\\ j\neq i}}^{M}\langle\|u_i(\boldsymbol{r}_i,t)\|^2\rangle\langle\|u_j(\boldsymbol{r}_j,t)\|^2\rangle\right\} \\ &= \langle M\rangle\langle\|u(\boldsymbol{r},t)\|^4\rangle \\ &\quad + 2\{\langle M\rangle\langle\|u(\boldsymbol{r},t)\|^2\rangle\}^2 \\ &= \langle N\rangle\iiint_{\Delta\Omega} d\boldsymbol{r}\langle\|u(\boldsymbol{r},t)\|^4\rangle_s \\ &\quad + 2\left\{\langle N\rangle\iiint_{\Delta\Omega} d\boldsymbol{r}\langle\|u(\boldsymbol{r},t)\|^2\rangle_s\right\}^2\end{aligned} \tag{5}$$

where we have assumed that the number of scatterers in $\Delta\Omega$ follows the Poisson distribution, and

$$\sum_{M=0}^{+\infty} P(M)M(M-1) = \langle M\rangle^2.$$

The ratio of the fourth moment to the square of the second moment is given by

$$\frac{\langle I^2(t)\rangle}{\langle I(t)\rangle^2} \approx 2\left(1+\frac{1}{\alpha}\right) \tag{6}$$

where

$$\frac{1}{\alpha} \equiv \frac{1}{2\langle N\rangle} \times \frac{\iiint_{\Delta\Omega} d\boldsymbol{r}\langle\|u(\boldsymbol{r},t)\|^4\rangle_s}{\left\{\iiint_{\Delta\Omega} d\boldsymbol{r}\langle\|u(\boldsymbol{r},t)\|^2\rangle_s\right\}^2}. \tag{7}$$

The quantity $1/\alpha$ is a measure of the deviation of the statistical properties of the echo signal from those of the Rayleigh distribution. When the average number of scatterers contributing to the echo signal is large, the statistical properties approach those of the Rayleigh distribution, where $1/\alpha \to 0$ and the ratio of the fourth moment to the square of the second moment $\to 2$.

From (2) and (7), $1/\alpha$ depends both on the measurement system and the medium. Insight into the latter dependency is gained by assuming conditions for which terms can be factored out of the integral in (2). For example, if each scattering function $\psi_i(\omega)$ is approximately independent of frequency over the bandwidth of the measurement system, we have

$$u_i(\boldsymbol{r}_i, t) \simeq \Psi_i(\omega_o) S_1(\boldsymbol{r}_i, t) \tag{8}$$

where ω_o is the center frequency of the system and

$$S_1(\boldsymbol{r}_i, t) \equiv \int_0^{+\infty} d\omega T(\omega) A_o(\omega) e^{-i\omega t} A^2(\boldsymbol{r}_i, \omega). \tag{8'}$$

Substituting (8) and (8′) into (7), we have

$$\frac{2}{\alpha} \simeq \frac{1}{N_{\text{eff}}(\omega_o)} \times \left(\frac{\iiint\limits_{\Delta\Omega} d\boldsymbol{r} \langle \|S_1(\boldsymbol{r}, t)\|^4 \rangle}{\left\{ \iiint\limits_{\Delta\Omega} d\boldsymbol{r} \langle \|S_1(\boldsymbol{r}, t)\|^2 \rangle \right\}^2} \right) \tag{9}$$

for this special case. $N_{\text{eff}}(\omega_o)$ is referred as an "effective scatterer number" density; it was defined previously [6] as,

$$N_{\text{eff}}(\omega_o) \equiv \langle N \rangle \times \frac{\langle \|\Psi(\omega_o)\|^2 \rangle_s^2}{\langle \|\Psi(\omega_o)\| \rangle_s^4}. \tag{10}$$

Equation (6) is essentially the same as (7) in Weng and Reid [5], who assume the echo signal statistics follow a k-distribution as derived by Jakeman [10]. Weng and Reid refer to α as a "clustering parameter" in the k-distribution. Experimentally they find it has a "linear relation with the log-scaled scatterer concentration." Our derivation shows that this parameter is related in a complicated way to both characteristics of the measurement system as well as to the properties of the scattering medium and that it is proportional to the scatterer number density. The latter could, of course, be dominated by large scatterers, if present, perhaps fulfilling the embodiment of a cluster for specific conditions.

B. Non-Gaussian Properties of the Time Domain Echo Signal Voltage

Alternative uses of statistical moments of ultrasound echo signals have focused on the time-domain echo signal [4], [11] rather than the complex amplitude. We derive an expression for these moments, assuming that the experimental conditions are the same as in the previous dicussion. From (1), the real echo signal voltage at time t is given by

$$V(t) = \sum_{i=1}^{M} v_i(\boldsymbol{r}_i, t) \tag{11}$$

where $v_i(\boldsymbol{r}_i, t) \equiv \mathrm{Re}\, u_i(\boldsymbol{r}_i, t)$, Re means "the real part of" the quantity following and, again, the sum is over all scatterers which contribute to the echo signal at time t. $v_i(\boldsymbol{r}_i, t)$ is a real, random variable with a zero ensemble average.

Using methods similar to those of the previous section, we find that the second and fourth moments of the echo signal voltage are given by

$$\begin{aligned} \langle J(t) \rangle &\equiv \langle V^2(t) \rangle \\ &= \sum_{M=0}^{+\infty} P(M) \left\langle \left\{ \sum_{i=1}^{M} \sum_{j=1}^{M} v_i(\boldsymbol{r}_i, t) v_j(\boldsymbol{r}_j, t) \right\} \right\rangle \\ &\approx \sum_{M=0}^{+\infty} P(M) \left\{ \sum_{i=1}^{M} \langle v_i^2(\boldsymbol{r}_i, t) \rangle \right\} \\ &= \langle M \rangle \langle v^2(\boldsymbol{r}, t) \rangle \\ &= \langle N \rangle \iiint\limits_{\Delta\Omega} d\boldsymbol{r} \langle v^2(\boldsymbol{r}, t) \rangle_s \end{aligned} \tag{12}$$

and

$$\begin{aligned} \langle J^2(t) \rangle &\equiv \langle V^4(t) \rangle \\ &= \sum_{M=0}^{+\infty} P(M) \\ &\cdot \left\{ \sum_{i=1}^{M} \sum_{j=1}^{M} \sum_{k=1}^{M} \sum_{l=1}^{M} \langle v_i(\boldsymbol{r}_i, t) v_j(\boldsymbol{r}_j, t) v_k(\boldsymbol{r}_k, t) v_l(\boldsymbol{r}_l, t) \rangle \right\} \\ &\approx \sum_{M=0}^{+\infty} P(M) \left\{ \sum_{i=1}^{M} \langle v_i^4(\boldsymbol{r}_i, t) \rangle \right. \\ &\quad \left. + 3 \sum_{i=1}^{M} \sum_{\substack{j=1 \\ j \neq i}}^{M} \langle v_i^2(\boldsymbol{r}_i, t) \rangle \langle v_j^2(\boldsymbol{r}_j, t) \rangle \right\} \\ &= \langle M \rangle \langle v^4(\boldsymbol{r}, t) \rangle + 3 \{ \langle M \rangle \langle v^2(\boldsymbol{r}, t) \rangle \}^2 \\ &= \langle N \rangle \iiint\limits_{\Delta\Omega} d\boldsymbol{r} \langle v^4(\boldsymbol{r}, t) \rangle_s \\ &\quad + 3 \left\{ \langle N \rangle \iiint\limits_{\Delta\Omega} d\boldsymbol{r} \langle v^2(\boldsymbol{r}, t) \rangle_s \right\}^2. \end{aligned} \tag{13}$$

In deriving (13) we have used the fact that the contributions of terms contributing to $\langle v_i^m(\boldsymbol{r}, t) \rangle$ are negligible when $m = 2n + 1$. The summation results in M terms for which $i = j = k = l$ and $3M(M-1)$ involving $i = j \neq k = l, i = l \neq k = j, i = k \neq j = l$.

Thus, the ratio of the fourth moment to the square of the second moment of the RF signal is given by

$$\frac{\langle J^2(t) \rangle}{\langle J(t) \rangle^2} \approx 3\left(1 + \frac{1}{\beta}\right) \tag{14}$$

where

$$\frac{1}{\beta} \equiv \frac{1}{3\langle N\rangle} \times \frac{\iiint\limits_{\Delta\Omega} d\boldsymbol{r}\langle v^4(r,t)\rangle_s}{\left\{\iiint\limits_{\Delta\Omega} d\boldsymbol{r}\langle v^2(\boldsymbol{r},t)\rangle_s\right\}^2} \tag{15}$$

is a measure of the deviation of this ratio from that which applies for Gaussian statistics. For large $\langle N\rangle, 1/\beta \to 0$.

Similar to section II-A, if each scattering function $\psi_i(\omega)$ is approximately independent of the ultrasound frequency over the bandwidth of the measurement system, we have

$$\frac{3}{\beta} \simeq \frac{1}{N_{\text{eff}}(\omega_o)} \times \left(\frac{\iiint\limits_{\Delta\Omega} d\boldsymbol{r}\langle \|S_2(\boldsymbol{r},t)\|^4\rangle}{\left\{\iiint\limits_{\Delta\Omega} d\boldsymbol{r}\langle \|S_2(\boldsymbol{r},t)\|^2\rangle\right\}^2}\right) \tag{16}$$

where

$$S_2(\boldsymbol{r}_i,t) \equiv \operatorname{Re}\int_0^{+\infty} d\omega T(\omega)A_o(\omega)e^{-i\omega t}A^2(\boldsymbol{r}_i,\omega) \tag{16$'$}$$

and $N_{\text{eff}}(\omega_o)$ is the effective scatterer number density. If (16) is solved for $\langle N\rangle$, the expression is analogous to that used by Sleefe and Lele [4] to estimate the scatterer number density.

C. The Relationship Between α and β

As suggested by Denbigh and Smith [3], the relationship between the parameters α and β can be derived. For band-limited pulses, we have

$$u(\boldsymbol{r},t) \approx a(\boldsymbol{r},t)e^{i(\phi(\boldsymbol{r},t)-2\omega_0 t)}$$

and

$$v(\boldsymbol{r},t) \approx a(\boldsymbol{r},t)\cos(\phi(\boldsymbol{r},t) - 2\omega_o t)$$

where $a(\boldsymbol{r},t)$ is a real function dependent on both the angular distribution function of a scatterer located at $\boldsymbol{r}$ and the ultrasound pressure amplitude at that position, while $\phi(\boldsymbol{r},t)$ is a phase factor that also depends on $\boldsymbol{r}$ and ω_o is the central frequency of the pulse.

For scatterers located in the far field of a flat circular disk transducer, or in the focal region of a focused transducer $a(\boldsymbol{r},t)$ depends mainly on the distance from the beam axis and the scattering function, while $\phi(\boldsymbol{r},t)$ depends mainly on the axial distance. Thus it can be assumed that $a(\boldsymbol{r},t)$ and $\phi(\boldsymbol{r},t)$ are random variables which are statistically independent and $\phi(\boldsymbol{r},t)$ can be regarded as being uniformly distributed over 2π radians. In that case, we have

$$\begin{aligned}\frac{\beta}{\alpha} &= \frac{3}{2} \times \frac{\iiint\limits_{\Delta\Omega} d\boldsymbol{r}\langle \|u(\boldsymbol{r},t)\|^4\rangle_s}{\left\{\iiint\limits_{\Delta\Omega} d\boldsymbol{r}\langle \|u(\boldsymbol{r},t)\|^2\rangle_s\right\}^2} \times \frac{\left\{\iiint\limits_{\Delta\Omega} d\boldsymbol{r}\langle v^2(\boldsymbol{r},t)\rangle_s\right\}^2}{\iiint\limits_{\Delta\Omega} d\boldsymbol{r}\langle v^4(\boldsymbol{r},t)\rangle_s} \\ &\approx \frac{3}{2} \times \frac{\int_0^{2\pi} d\phi}{\left\{\int_0^{2\pi} d\phi\right\}^2} \times \frac{\left\{\int_0^{2\pi} d\phi \cos^2\phi\right\}^2}{\int_0^{2\pi} d\phi\cos^4\phi} \\ &= 1.\end{aligned} \tag{17}$$

Thus, we see that there is a simple relationship between the "non-Rayleigh parameter", α derived from the magnitude of the complex echo signal and the "non-Gaussian parameter", β derived from the real RF echo signal voltage. They contain essentially the same information about the scattering medium.

III. Experimental Verification

The relationship between α and β was verified by measuring the scattered echo signals from three phantoms, each containing a different scatterer number density. Phantoms consist of agar gel, with $73 \pm 5\,\mu$m glass beads as scatterers. The speed of sound in each phantom is 1555 m/s and the attenuation coefficient is 0.068 dB/cm/MHz. They were placed in water in the focal region of either of two broad-band ultrasound transducers. One transducer is 3.5 MHz, has a 19.2 mm aperture and a radius of curvature of 9.65 cm, while the other transducer is 5.0 MHz with an 18.6 mm aperture and a radius of curvature of 8.5 cm. Each transducer was excited with single cycle pulses at its center frequency (3.5 and 5.0 MHz, respectively).

The resultant time domain echo signals were truncated using a 5.0 μs rectangular gate. They were recorded on a digital oscilloscope and stored in a computer for off-line analysis. The sampling frequency of the oscilloscope was 100 MHz. For each transducer and each phantom echo signals were recorded for 40 different locations, realized by translating the sample perpendicularly to the transducer axis. Each translation step was 4.0 mm. Since the full width at half-minimum (FWHM) of the beam was less than the translation steps in the interrogated region, successive waveforms were uncorrelated.

The square magnitude of the complex echo signal was found by applying a Hilbert transform to each gated waveform and then summing the squared real and imaginary parts. The moments of the signal envelope were then computed using

$$\langle X^k\rangle \simeq \frac{\sum_{i=1}^{n}\sum_{j=1}^{m} X_{i,j}^k}{nm} \tag{18}$$

where n is the number of waveforms and m is the number of sample points wthin the time gate function. The parameter $2/\alpha$ was then computed using (6).

The moments of the RF echo signal were calculated in a similar fashion, only now the actual sampled signal values recorded by the digital oscilloscope were substituted into

TABLE I
EXPERIMENTAL VALUES FOR THE NON-RAYLEIGH PARAMETER α AND THE NON-GAUSSIAN PARAMETER β FOR DIFFERENT SCATTERER CONCENTRATIONS. IN EACH CASE THE RATIO α/β IS EQUAL TO 1, AGREEING WITH THE THEORETICAL PREDICTION

Transducers	3.5 MHz			5.0 MHz		
Phantoms	134/cc	400/cc	750/cc	134/cc	400/cc	750/cc
$2/\alpha$	2.79 ± 0.43	1.05 ± 0.18	0.57 ± 0.10	9.26 ± 0.33	3.23 ± 0.39	1.66 ± 0.21
α	0.72 ± 0.11	1.90 ± 0.33	3.51 ± 0.62	0.22 ± 0.01	0.62 ± 0.07	1.20 ± 0.15
$3/\beta$	4.14 ± 0.65	1.56 ± 0.27	0.83 ± 0.15	13.9 ± 1.4	4.80 ± 0.58	2.47 ± 0.34
β	0.72 ± 0.11	1.92 ± 0.33	3.61 ± 0.65	0.22 ± 0.02	0.62 ± 0.08	1.21 ± 0.16
α/β	1.00 ± 0.33	0.99 ± 0.39	0.97 ± 0.36	1.00 ± 0.20	1.00 ± 0.19	0.99 ± 0.23

TABLE II
VALUES OF α AND β VERSUS AXIAL DISTANCE OF THE CENTER OF THE SAMPLE VOLUME FROM THE FOCAL POINT OF THE TRANSDUCER, DEMONSTRATING A BEAM WIDTH EFFECT. THE 400/CC PHANTOM AND 5.0 MHz TRANSDUCER WERE USED

Distance from Focal Point	0.0 mm	7.7 mm	15.4 mm	23.1 mm
$2/\alpha$	3.23 ± 0.39	2.85 ± 0.33	1.44 ± 0.35	0.73 ± 0.20
α	0.62 ± 0.07	0.70 ± 0.08	1.39 ± 0.34	2.74 ± 0.75
$3/\beta$	4.80 ± 0.58	4.28 ± 0.50	2.16 ± 0.53	1.09 ± 0.30
β	0.62 ± 0.08	0.70 ± 0.08	1.39 ± 0.34	2.75 ± 0.76
α/β	1.00 ± 0.18	1.00 ± 0.16	1.00 ± 0.34	1.00 ± 0.39

the above expression. $3/\beta$ was then determined for each experimental situation, i.e., each phantom and transducer.

Results are summarized on Table I. The parameters α and β are listed for each phantom for both the 3.5 and 5.0 MHz focused transducers. Results presented are means and standard deviations for nine sets of repeated measurements. The last row in Table I presents the ratio of α/β for each experiment. This ratio is equal to 1.0, agreeing with the theoretical prediction in (17).

For fixed experimental conditions, both α and β are found to be proportional to the scatterer number density, as expected. For example, when the scatterer number density is increased from 400/cc to 750/cc (x1.87), α increases by a factor of 1.85 for the 3.5 MHz experimental data and 1.94 for the 5.0 MHz data. These results also show that α and β are related to the transducer beam cross-sectional area. Values of α and β obtained using the 5.0 MHz transducer are lower than corresponding values obtained using the 3.5 MHz transducer due to the greater beam cross-sectional area of the latter transducer. The standard deviations in Table I originate mainly from statistical fluctuations in the echo signals and the resultant moments. Because the statistical uncertainty is related to the average number of scatterers contributing to the echo signal [12], the standard deviations of α and β are also lower for the 5.0 MHz transducer data than the 3.5 MHz data. For either transducer, because the beam area in the focal region is smaller than the beam area in other regions, the values of α and β for the focal region are always smaller than those from other regions. This is shown in Table II.

IV. DISCUSSION AND CONCLUSION

Theoretical expressions for the non-Rayleigh and the non-Gaussian parameters applied to signals from a medium containing randomly distributed scatterers are derived in this paper. These parameters are related both to the properties of the measurement system, including the system frequency response and the transducer ultrasound field, as well as to the insonified medium. The latter dependency is expressed here in terms of a scatterer number density and moments of the scattering function. At low scatterer concentrations, if the scatterers are independent and randomly distributed, α and β are proportional to the scatterer number density.

In contrast to these results, Weng and Reid [5] found experimentally a non-Rayleigh parameter proportional to the log of the scatterer number density. Their measurements were made for samples having greater scatterer number densities than ours, ranging from 2×10^3 cm^{-3} to 6×10^4 cm^{-3}. We do not know the reason for the difference between those authors' results and ours, but note that other factors, such as particle clustering, volume coherent effects or changes in attenuation between phantoms with low scatter number density and phantoms with higher scatterer number density, may be responsible. Attenuation of higher frequency components of a pulsed beam has the effect of broadening the beam at points distal to the focal region [13], and the α parameter (or the β parameter) is sensitive to beam width. For our samples, the attenuation in all three phantoms was the same. However, attenuation was not reported by Weng and Reid [5].

When the scatterer concentration is high, the statistical properties of the envelope of the echo signal approach those of a Rayleigh distribution, while the statistical properties of the RF signal approach those of a Gaussian distribution. Thus the ratio of the fourth moment to the square of the second moment of the RF signal goes to three, while the corresponding ratio for the signal envelope goes to two.

Under an assumption of band-limited ultrasonic pulses, a simple relationship between the non-Gaussian and the non-Rayleigh parameters was derived. Experimental results presented in this paper confirmed this relationship. This suggests that for this type of analysis, it is possible to get the same useful information regarding a scattering medium from both the envelope and the RF echo signal. Conceivably, the envelope statistics could be derived from conventional ultrasound B-mode images.

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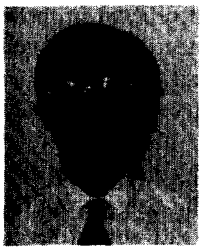

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