



## Improved algorithm for estimation of attenuation along propagation path using backscattered echoes from multiple sources

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### ABSTRACT

Accurately determining the attenuation along the propagation path leading to a region of interest could significantly improve diagnostic ultrasound tissue characterization since tissue characterization requires exact compensation for the frequency-dependent attenuation along the propagation path. In a previous study (JASA, 124:1367, 2008), it was shown that the total attenuation can be determined by using the backscattered echoes from multiple sources. The preliminary computer simulation results, had an average error between  $-0.3$  and  $+0.2$  dB/MHz for the cases tested with a trend towards increasing error with increasing correlation length (i.e., characteristic size of the tissue microstructure of the scattering medium) and attenuation along the propagation path. Therefore, the goal of this study was to improve the accuracy of previously derived algorithm and reduce the dependence of the algorithm on correlation length and attenuation. In this study, the previous derivations were redone and the assumptions made by the algorithm regarding the scattering properties of the medium and the shape of the backscattered power spectrum were relaxed. The revised algorithm was then verified using computer simulations of five sources (6, 8, 10, 12, and 14 MHz, 50% bandwidth) exposing a homogeneous tissue region. The simulated tissue had microstructure following a Gaussian spatial correlation function (i.e.,  $\exp(-0.827(ka_{eff})^2)$  where  $k$  is the wavenumber) with effective radii,  $a_{eff}$ , of 5–55  $\mu\text{m}$  (one size per simulated case) placed at a density of  $250/\text{mm}^3$  ( $\sim 5$  scatterers/resolution cell for 14 MHz transducer). The attenuation of the tissue was also varied from 0.1 to 0.9 dB/cm-MHz. The computer simulations demonstrated that the modifications significantly improved the accuracy of the algorithm resulting in average errors between  $-0.04$  and 0.1 dB/MHz which is three times better than the error performance of the original algorithm.

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### 1. Introduction

Ultrasound tissue characterization has shown great potential in improving the accuracy of diagnostic medical ultrasound. For example, quantifying the characteristic correlation length and acoustic concentration of scatterers has been able to distinguish between fibroadenomas and carcinomas in implanted tumors in rats [1]. Other investigators have also used tissue characterization to quantify ocular tumors, blood, liver, muscle, myocardial injury, plaque, kidney, prostate, bone, skin, and tissue changes following therapy [2–16]. These techniques are based on quantifying the frequency dependence of the backscatter, and therefore require an accurate estimate of the frequency dependence of the attenuation along the propagation path in order to perform a valid assessment of the tissue. Furthermore much of the difficulty in extending the promising results of tissue characterization to the clinic can be attributed to the lack of a robust algorithm to estimate the frequency dependence of the attenuation along the propagation path

as has been known for over 25 years [17]. For example, utilizing tissue characterization to assess ocular tumors has been successfully implemented in the clinic, despite competing technologies such as optical coherence tomography, and much of its success can be attributed to the propagation path being very similar to water which has a known frequency-dependent attenuation [10,17,18].

In addition to ultrasound tissue characterization, an knowledge of the attenuation along the propagation path would also be beneficial to other applications of medical ultrasound such as ultrasound therapy planning. Currently, high-intensity focused ultrasound (HIFU) is being developed to thermally ablate tumors and other tissue noninvasively [19–21]. In addition to thermal coagulation, the use of ultrasound in various other applications such as drug delivery, opening the blood–brain barrier, destroying biofilms on medical implants, and mechanically destroying or removing tissue in a controlled fashion (i.e., ultrasound histotripsy) is also being explored [22–29]. In order to maximize the effectiveness of these therapeutic applications in the clinic, it is likely that careful therapy planning will be needed to determine which patients are the best candidates and to design the exposure. Such

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planning is common practice in radiation treatments for cancer. Knowing the total attenuation along the propagation path leading to the targeted tissue region could significantly enhance the accuracy of the therapy planning by allowing the *in vivo* ultrasound intensity and pressures to be estimated *a priori*. Of course, such an estimate would also require proper inclusion of nonlinear propagation which is also an active area of research.

We recently developed a new algorithm for estimating the total attenuation along the propagation path by scanning the same tissue region with multiple ultrasound sources and comparing the down-shift in spectral peak frequency between the different sources [30]. Unlike previous algorithms, the new algorithm was able to obtain estimates of attenuation without making overly restrictive assumptions regarding the frequency dependence of the backscatter [31–33] or the homogeneity of the intervening tissue [34,35]. Unfortunately, the new algorithm had an average error between  $-0.3$  and  $+0.2$  dB/MHz for the cases tested with a trend towards increasing error as the scatterer correlation length (i.e., characteristic size of the tissue microstructure of the scattering medium as described in [36]) and/or the attenuation along the propagation path increased. Therefore, the goal of this study was to reduce the error and the observed dependence on scatterer correlation length and attenuation.

In this paper, we first re-derive the algorithm while relaxing some of the assumptions including an assumption regarding the scattering properties of the medium as will be detailed in the revised derivation. An assumption that the backscattered echoes have a Gaussian power spectrum that was made in the original algorithm is also relaxed by modifying the implementation of the algorithm. The revised algorithm is then verified using computer simulations of spherically-focused source exposing an infinite half-space. Lastly, conclusions are drawn based on how the improvement in accuracy might translate to better clinical diagnosis of benign and malignant tumors.

## 2. Revised derivation of algorithm

The backscattered power spectrum received by an ultrasound source from a random distribution of weak scatterers at the focus that satisfy the Born approximation is given by [30,37]

$$E[|V_{scat}(f)|^2] \propto k^4 |V_{plane}(f)|^2 A_{comp}(f) e^{-4\alpha_{tot}z_T} F_\gamma(f, a). \quad (1)$$

In Eq. (1),  $F_\gamma(f, a)$  is the form factor describing the scattering model for the tissue and depends on both frequency,  $f$ , and the effective scatterer radius,  $a$ , of the tissue. The total attenuation along the propagation path is given by  $\alpha_{tot}z_T$  where  $z_T$  is the distance from the aperture plane to the focal plane for the transducer, and  $\alpha_{tot}$  is the *effective* attenuation coefficient for the tissue along the propagation path and is weighted to account for tissue inhomogeneity in the intervening tissue layers between the source and the scattering region of interest (ROI). In our algorithm, we assume that  $\alpha_{tot}$  is approximately given by  $\alpha_{tot} = \alpha_o f$  and the goal of our algorithm is to find  $\alpha_o$ . Also,  $k$  is the wave number, and  $|V_{plane}(f)|^2$  is the backscattered power spectrum that would be returned from a rigid plane placed at the focal plane in a water bath and is obtained independently to calibrate the echoes from the tissue.

The remaining term in Eq. (1),  $A_{comp}(f)$ , is a generalized attenuation-compensation function that accounts for focusing and attenuation within the ROI and is given by

$$A_{comp}(f) = \int_{-L/2}^{L/2} ds_z \left( g_{win}(s_z) \exp\left(-\frac{4s_z^2}{w_z^2}\right) \exp(4\alpha_{loc}s_z) \right). \quad (2)$$

In Eq. (2),  $L$  is the length of the windowing function,  $g_{win}$ , in millimeters that is used to gate the backscattered waveform when selecting the ROI, and  $\alpha_{loc}$  is the local attenuation within the ROI.  $w_z$  is the

effective Gaussian depth of focus that results from approximating the field in the focal region with a Gaussian function and can be used to quantify the ultrasound field for both spherically focused and array sources.  $w_z$  depends linearly on wavelength and is given by  $w_z = 6.01\lambda(f\#)^2$  for an ideal spherically-focused source where  $f\#$  is the  $f$ -number of the source [37]. As has been shown previously, it is possible to accurately estimate  $A_{comp}(f)$  by first measuring  $w_z$  for the transducer and then approximating the local attenuation by  $\alpha_{loc} \equiv \sqrt{(\alpha_{high}^2 + \alpha_{low}^2)}/2$  where  $\alpha_{high}$  and  $\alpha_{low}$  are the largest and smallest attenuation values expected for the tissue [38]. Therefore, it is possible to compensate for attenuation and focusing within the ROI prior to implementing our algorithm yielding

$$E[|V_{scat}(f)|^2]_{compensated} \propto k^4 |V_{plane}(f)|^2 e^{-4\alpha_{tot}z_T} F_\gamma(f, a) \quad (3)$$

as was also discussed in our earlier publication [30]. Our earlier work also assumed that the backscattered spectrum was approximately Gaussian such that

$$k^4 |V_{plane}(f)|^2 \propto \exp\left(-\frac{(f-f_o)^2}{2\sigma_o^2}\right) \quad (4)$$

where  $f_o$  is the frequency corresponding to the spectral peak, and  $\sigma_o^2$  quantifies the bandwidth of the received echoes.

In order to develop the algorithm in our earlier work, we assumed that the form factor  $F_\gamma(f, a_{eff})$  was approximately given by

$$F_\gamma(f, a_{eff}) \propto \exp(-Af^n) \quad (5)$$

over the frequency range of interest where  $A$  is correlated with the effective scatterer size for the tissue and  $n \sim 2$  as has been assumed previously [33]. While clearly valid for the Gaussian form factor tissue model,  $F_{gauss}(ka) = \exp(-0.827(ka)^2)$ , that has been used extensively for soft tissue [1,5,7], the approximation given by (5) is also valid for a wide variety of other form factors as is illustrated in Fig. 1a for the fluid-filled sphere form factor,  $F_{fluid\_sphere}(ka) = \left[\frac{j_1(2ka)}{(2/3)ka}\right]^2$ , and Fig. 1b for the spherical shell form factor,  $F_{shell}(ka) = [j_0(2ka)]^2$ . The fluid-filled sphere has been used in the past to model soft tissue (like the Gaussian form factor model) while the spherical shell has been used to describe scattering from glass beads in tissue mimicking phantoms. The solid line in these plots is the true value of the form factor while the dashed line is the best fit that was obtained by fitting a function of the form  $A(ka)^n$  to  $-\ln(F_{fluid/sphere}(ka))$  and  $-\ln(F_{shell}(ka))$  for Fig. 1a and b respectively. The  $ka$  range used to fit the fluid-filled sphere form factor was 0.01 to 1.2 while the  $ka$  range used for the shell was

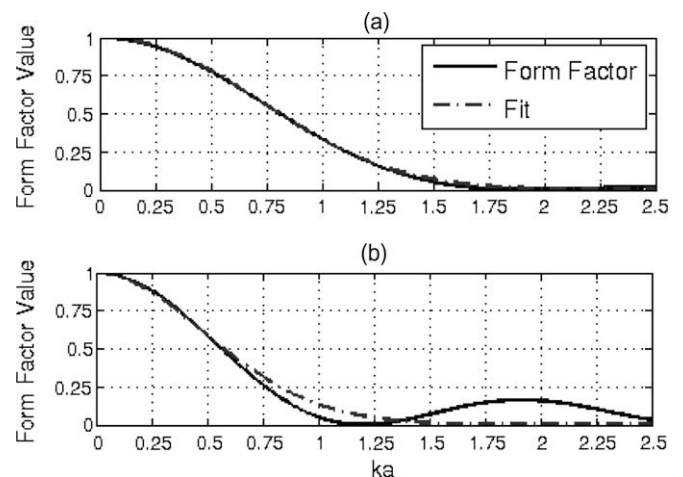


Fig. 1. Form factor with fit of the form  $\exp(-A(ka)^n)$  for: (a) fluid-filled sphere and (b) spherical shell.

0.01–0.6. A smaller  $ka$  range was used for the shell form factor due to the resonance at a  $ka$  of 1.2 impacting the fit. A similar resonance has not been observed in soft tissue, so this restriction on the  $ka$  range would only be important when verifying the algorithm using phantoms and would not be applied clinically. The fit of the fluid-filled sphere form factor was limited to 1.2  $ka$  because values less than 1.2 are of the greatest interest when quantifying the tissue microstructure [39]. The best fit yielded

$$\begin{aligned} F_{\text{fluid\_sphere}}(ka) &\cong \exp(-1.11(ka)^{2.167}) \\ F_{\text{shell}}(ka) &\cong \exp(-2(ka)^{1.914}) \end{aligned} \quad (6)$$

Clearly, the approximation of  $n \sim 2$  is appropriate for most form factors over the  $ka$  range of interest. Therefore, unlike the Spectral Fit Algorithm [31–33], the new algorithm has only a weak dependence on the tissue model assumed for the scattering since all three models have approximately the same frequency dependence.

After making the approximations given in (4) and (5), the impact of scattering and attenuation along the propagation path was incorporated as a series of Gaussian transformations on the Gaussian power spectrum yielding

$$E[|V_{\text{scat}}(f)|^2]_{\text{compensated}} \propto \exp\left(-\frac{(f - \tilde{f}_o'')^2}{2\tilde{\sigma}_\omega^2}\right). \quad (7)$$

In this equation,  $\tilde{f}_o''$  is the expected spectral peak frequency, and  $\tilde{\sigma}_\omega^2$  is the new bandwidth of the backscattered spectrum.  $\tilde{f}_o''$  and  $\tilde{\sigma}_\omega^2$  were related to the peak frequency and bandwidth of the echoes returned from a rigid plane placed at the focal plane (i.e.,  $f_o$  and  $\sigma_\omega^2$ ) in our earlier work [30] by

$$f_o - \tilde{f}_o'' \cong 4z_T \alpha_o \left( \frac{\sigma_\omega^2 + \tilde{\sigma}_\omega^2}{2} - \frac{\sigma_\omega^2 \tilde{\sigma}_\omega^2 An}{2} \right) + \tilde{\sigma}_\omega^2 An f_o \quad (8)$$

and

$$\tilde{\sigma}_\omega^2 \cong \frac{\sigma_\omega^2}{1 + An\sigma_\omega^2} \quad (9)$$

when  $n \sim 2$ .

In our earlier derivations, the term  $\frac{\sigma_\omega^2 \tilde{\sigma}_\omega^2 An}{2}$  from (8) was assumed to be small compared to  $\frac{\sigma_\omega^2 + \tilde{\sigma}_\omega^2}{2}$  and hence was neglected in the rest of the derivation. Since publishing our earlier paper [30], we have determined that neglecting this term resulted in a slight bias in the total attenuation estimates generated by our algorithm (average error between  $-0.3$  and  $+0.2$  dB/MHz) that increased with larger scatterer sizes. Therefore, in this paper, the derivation will be repeated and the term  $\frac{\sigma_\omega^2 \tilde{\sigma}_\omega^2 An}{2}$  will not be neglected. Instead, we will assume that  $An\sigma_\omega^2 \ll 1$  and therefore keep only first order terms in  $An\sigma_\omega^2$ . Under this new relaxed approximation, (9) becomes  $\tilde{\sigma}_\omega^2 \cong \sigma_\omega^2 - An\sigma_\omega^4$  allowing (8) to be written as

$$f_o - \tilde{f}_o'' \cong 4z_T \alpha_o \tilde{\sigma}_\omega^2 + \tilde{\sigma}_\omega^2 An f_o \quad (10)$$

when we neglect higher order terms in  $An\sigma_\omega^4$ . Dividing (10) by  $4z_T \tilde{\sigma}_\omega^2$  yields the normalized downshift in peak frequency relative to the echoes from the rigid plane placed at the focal plane given by

$$\xi(f_o) = \frac{f_o - \tilde{f}_o''}{4z_T \tilde{\sigma}_\omega^2} \cong \alpha_o + \left( \frac{An}{4z_T} \right) f_o. \quad (11)$$

Measuring  $\xi(f_o)$  from the backscattered echoes from a ROI using multiple ultrasound transducers (i.e., different  $f_o$ ), fitting a line to  $\xi(f_o)$  versus  $f_o$ , and finding the intercept of that line yields the slope of the total attenuation along the propagation path,  $\alpha_o$ . Theoretically, (11) will yield  $\alpha_o$  even for inhomogeneous tissues while only requiring echoes from a specific region of interest.

In addition to improving the robustness of our algorithm in the presence of larger scatterer sizes, we also sought to relax some of

the assumptions regarding the spectral shape. Recall that our new algorithm assumed that the backscattered power spectrum from tissue was approximately given by a Gaussian function. However, this approximation is typically only valid for a small range of frequencies about the spectral peak frequency and varies for different transducers and excitation conditions. As a result, the assumption tends to breakdown introducing a bias into the estimates as the as the spectral peak for the tissue gets farther away from the spectral peak for the reference (i.e., difference between  $f_o$  and  $\tilde{f}_o''$  increases due to large attenuation values). In order to avoid this problem and relax the Gaussian approximation for the spectrum, we decided to first find an approximate value for the total attenuation slope, termed  $\alpha_{\text{guess}}$ , by

$$\alpha_{\text{guess}} = \text{mean}_{f_o} \{ \xi(f_o) \} \quad (12)$$

where  $\xi(f_o)$  is given by (11).  $\alpha_{\text{guess}}$  would be the attenuation slope under purely Rayleigh scattering (i.e., point scatterers) and therefore is an overestimate of  $\alpha_o$ . Once  $\alpha_{\text{guess}}$  is determined, we calculate a new reference spectrum for each source given by

$$|V_{\text{ref}}(f)|^2 = \exp(-4\alpha_{\text{guess}} z_T f) k^4 |V_{\text{plane}}(f)|^2. \quad (13)$$

The new reference spectra peak at approximately the same location as the backscattered spectra from the ROI reducing the frequency range over which the Gaussian approximation for the spectral shape needs to be valid. After finding the new reference spectra, we find the new spectral peak value for each reference,  $f_o'$ , and calculate a new value for  $\xi(f_o') = (f_o' - \tilde{f}_o'')/4z_T \tilde{\sigma}_\omega^2$ .  $\xi(f_o')$  corresponds to

$$\xi(f_o') \cong (\alpha_o - \alpha_{\text{guess}}) + \left( \frac{An}{4z_T} \right) f_o' \quad (14)$$

allowing us to determine  $\alpha_o$  by finding the intercept of the line through  $\xi(f_o')$  and then adding the value of  $\alpha_{\text{guess}}$  to the intercept value.

### 3. Computer simulation results validating revised algorithm

In order to evaluate the revised algorithm, computer simulations were performed. In the simulations, five spherically focused ultrasound sources exposed exactly the same homogeneous attenuating half-spaces. Spherically-focused sources were selected in this initial validation so that a rigid plane placed at the focal plane could be used as reference target. A planar target was desired to simplify our preliminary analysis. However, other methods for quantifying the acoustic field, such as a reference phantom, will be explored in the future. The sources used in the simulations had focal lengths of 5 cm and  $f$ -numbers of 4. The center frequencies of the sources were 6, 8, 10, 12, and 14 MHz, and each source had a  $-3$  dB transmitted bandwidth of 50%. These frequencies were selected to capture both the high end of the current diagnostic transducers as well as bridge the gap to some of the higher frequency transducers that have been utilized in tissue characterization experiments.

The sound speed of the half-spaces was 1540 m/s, and the attenuation of the half-spaces was varied from 0.1 dB/cm-MHz to 0.9 dB/cm-MHz in order to assess the dependence of the algorithm on attenuation. These values for attenuation were selected to span the range of most soft tissues in clinical applications. Likewise, a sound speed value of 1540 m/s was selected since this is a reasonable value for most tissue. Only one value of sound speed was used because the algorithm has little dependence on the speed of sound in the tissue provided the location of the focal zone can be approximately determined. The scattering structures within each half-space had Gaussian correlation functions (i.e., form factor of  $F_\gamma(f, a_{\text{eff}}) = \exp(-0.827(ka_{\text{eff}})^2)$ ) with an  $a_{\text{eff}}$  from 5 to 55  $\mu\text{m}$  and

were positioned at a density of 250/mm<sup>3</sup> (~5 scatterers per resolution cell for 14 MHz transducer). Every simulated half-space had only a single scatterer size and a constant value of attenuation. We selected our range for scatterer sizes to span the  $a_{eff}$  determined for fibroadenomas ( $52.5 \pm 7.0 \mu\text{m}$ ) and carcinomas ( $14.0 \pm 2.3 \mu\text{m}$  using 20 MHz transducer,  $19.9 \pm 3.2 \mu\text{m}$  using 8.5 MHz transducer) in previous animal experiments [1].

For each half-space, 250 independent three dimensional distributions of scatterers were generated and grouped into sets of 25 yielding 10 independent data sets. The backscattered echoes from each distribution using each source (five different sources) were then simulated, and windowed with a rectangular windowing function in the time domain having a length corresponding to 4.36 mm in tissue depth. The window length of 4.36 mm and the grouping of 25 distributions per set were found to be optimal in our earlier implementation of the algorithm [30]. A rectangular windowing function has previously been shown to yield the best performance when compensating the backscattered power spectrum for windowing, focusing, and local attenuation [38]. After windowing, the power spectra were calculated for all 25 scatterer distributions for each source in each set. The power spectra were then averaged within each set to yield estimates of  $E[|V_{scat}(f)|^2]_{compensated}$  for each source after correcting for focusing and attenuation in the ROI. These estimates for  $E[|V_{scat}(f)|^2]_{compensated}$  for each source could then be used to obtain 10 estimates of  $\alpha_o$ , one per independent set, for every half-space using our algorithm.

The results using our modified algorithm are summarized in Figs. 2 and 3. In these plots, the symbols represent the average error in the attenuation estimates found for each half-space while the error bars correspond to one standard deviation of the error in the attenuation estimates for each half-space. The figures show the errors in the attenuation estimates both in percent and in dB/MHz as calculated by

$$Error_{\%} = 100 \cdot \frac{(\alpha_o|_{Estimated} - \alpha_o|_{True})}{\alpha_o|_{True}} \quad (15)$$

$$Error_{dB/MHz} = (\alpha_o|_{Estimated} - \alpha_o|_{True}) \cdot Z_T$$

where  $\alpha_o|_{Estimated}$  is the estimated value for the attenuation slope found by our algorithm and  $\alpha_o|_{True}$  is the true value for the attenuation slope for the half-space. The error in percent is more applicable to therapy planning applications while the error in dB/MHz is more relevant in tissue characterization applications [31–33]. Also, recall that  $Z_T$  is the propagation distance into the tissue therefore the error

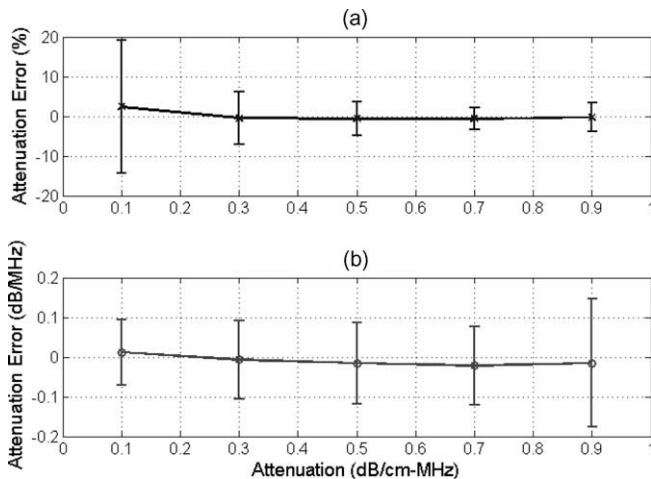


Fig. 2. (a) % error and (b) error in dB/MHz in the attenuation estimate for half-space attenuations from 0.1 to 0.9 dB/cm-MHz for a scatterer size of 25  $\mu\text{m}$  using the modified algorithm developed in this paper. The error bars correspond to one standard deviation above and below the mean value.

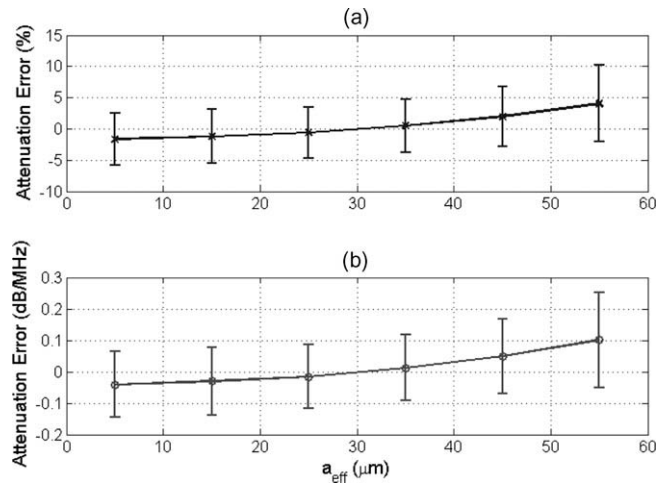


Fig. 3. (a) % error and (b) error in dB/MHz in the attenuation estimate for a half-space attenuation of 0.5 dB/cm-MHz for scatterer sizes from 5 to 55  $\mu\text{m}$  using the modified algorithm developed in this paper. The error bars correspond to one standard deviation above and below the mean value.

in dB/MHz quantifies the absolute error in the attenuation estimates and can be generalized to different focusing configurations.

When changing the attenuation of the half-space as shown in Fig. 2, the average error in the attenuation estimates varies from  $-0.02$  dB/MHz for a half-space attenuation of 0.7 dB/cm-MHz to 0.01 dB/MHz for a half-space attenuation of 0.1 dB/cm-MHz with no trend with half-space attenuation distinguishable. These results are considerably better than the results obtained using our original algorithm (see Fig. 4b published in [30]) where the average error in the attenuation estimates monotonically changed from  $-0.02$  to  $-0.3$  dB/MHz as the half-space attenuation changed from 0.1 to 0.9 dB/cm-MHz. Therefore, using our revised algorithm, the accuracy of our estimates (i.e., average error) are clearly independent of the half-space attenuation (Fig. 2) and yield accurate estimates as the attenuation is increased from 0.1 to 0.9 dB/cm-MHz ( $x$ -axis on plot) unlike our original algorithm [30].

The performance of the new algorithm as a function of varying correlation length is also considerably better than our original algorithm. For the new algorithm, shown in Fig. 3, the average error in the attenuation estimates increases monotonically with scatterer size from  $-0.04$  dB/MHz for a scatterer size of 5  $\mu\text{m}$  to

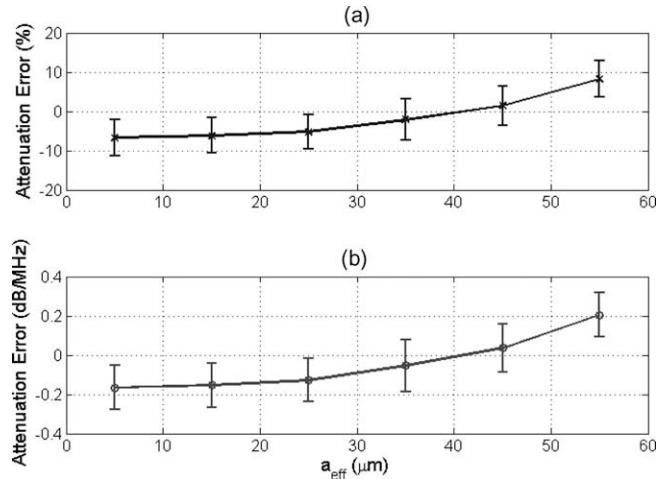


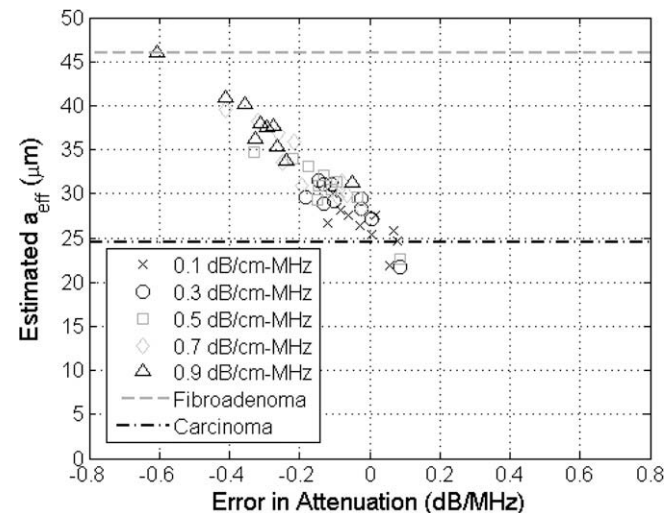
Fig. 4. (a) % error and (b) error in dB/MHz in the attenuation estimate for a half-space attenuation of 0.5 dB/cm-MHz for scatterer sizes from 5 to 55  $\mu\text{m}$  using the original algorithm described in [30]. The error bars correspond to one standard deviation above and below the mean value.

+0.1 dB/MHz for a scatterer size of 55  $\mu\text{m}$ . Whereas, for the original algorithm [30], shown in Fig. 4, the average error in the attenuation estimates increases from  $-0.17$  to  $+0.2$  dB/MHz as the scatterer size increases from 5 to 55  $\mu\text{m}$ . Therefore, the accuracy of the revised algorithm was over twice as good as the original algorithm with a much weaker dependence on scatterer size. There is still some dependence because the revised algorithm still assumes that  $An\sigma_{\omega}^2 \ll 1$  where  $A$  is proportional to the square of the correlation length,  $a_{\text{eff}}$ .

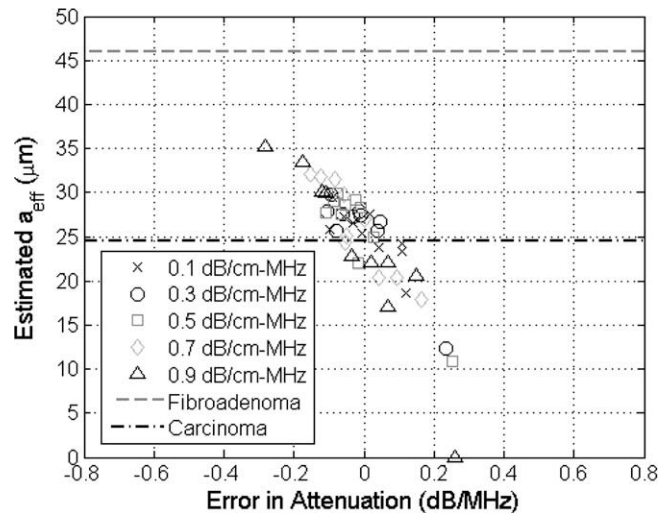
#### 4. Discussion/conclusions

In this paper, we improved the accuracy of our earlier algorithm [30] by relaxing the approximations regarding scatterer size and the Gaussian shape of the power spectrum. The accuracy and precision of our new algorithm was then assessed by computer simulations with varying scatterer sizes ( $a_{\text{eff}}$  of 5–55  $\mu\text{m}$ ) and attenuations (0.1–0.9 dB/cm-MHz). The magnitude of the largest average error in the attenuation estimates,  $|Error_{\text{dB/MHz}}|$ , was 0.1 dB/MHz for new algorithm which was three times smaller than the largest error of 0.3 dB/MHz found using our original algorithm. Therefore, the performance of the revised algorithm is considerably better than the performance of the original algorithm.

Before concluding, it is important that the improvement in the accuracy of the attenuation estimates obtained by the revised algorithm be connected to the future potential clinical impact of the work. To assist with this discussion, the attenuation estimates obtained using both our revised and our original algorithm were used to obtain estimates of  $a_{\text{eff}}$  as has been done in numerous prior publications [1,36–38]. In every case, the same echoes that were used to obtain an attenuation estimate were reprocessed using that estimated attenuation to generate a value for the scatterer size. This was done so that the variations in the attenuation estimates would be correctly translated to variations in the scatterer size estimates. The results for the half-spaces with a scatter size of 25  $\mu\text{m}$  and attenuations from 0.1 to 0.9 dB/cm-MHz using the echo data from the simulated 8 MHz source are shown in Figs. 5 and 6 for the original and revised algorithm respectively. Data from the 8 MHz source was selected because it would be comparable to earlier experimental data obtained using an 8.5 MHz source aimed at distinguishing between fibroadenomas and carcinomas in animal



**Fig. 5.** Scatter plot of showing estimated scatter size in  $\mu\text{m}$  versus the corresponding error in the attenuation estimate in dB/MHz for half-space attenuations from 0.1 to 0.9 dB/cm-MHz (different symbols) when the true scatterer size was 25  $\mu\text{m}$  using the original algorithm described in [30]. The lines on the plots correspond to the largest scatterer size measured for carcinomas (24.5  $\mu\text{m}$ ) and the smallest scatterer size measured for fibroadenomas (46  $\mu\text{m}$ ) in animal experiments reported in [1].



**Fig. 6.** Scatter plot of showing estimated scatter size in  $\mu\text{m}$  versus the corresponding error in the attenuation estimate in dB/MHz for half-space attenuations from 0.1 to 0.9 dB/cm-MHz (different symbols) when the true scatterer size was 25  $\mu\text{m}$  using the modified algorithm developed in this paper. The lines on the plots correspond to the largest scatterer size measured for carcinomas (24.5  $\mu\text{m}$ ) and the smallest scatterer size measured for fibroadenomas (46  $\mu\text{m}$ ) in animal experiments reported in [1].

experiments [1]. Each symbol in the plots corresponds to an attenuation estimate and the corresponding  $a_{\text{eff}}$  estimate from one of the 10 independent data sets generated for each half-space and used to evaluate the algorithm previously. The largest scatterer size measured for the carcinomas of 24.5  $\mu\text{m}$  and the smallest scatterer size measured for the fibroadenomas of 46  $\mu\text{m}$  as identified from Fig. 7 in [1] are also identified as lines in our figures.

From Figs. 5 and 6, it is clear that the scatterer size is underestimated when the attenuation of the half-space was overestimated (i.e., positive value of attenuation error) and overestimated when the attenuation of the half-space was underestimated (i.e., negative value of attenuation error) as has also been observed previously [32,33]. In addition, some of the scatterer size estimates using the original algorithm (Fig. 5), which should have been 25  $\mu\text{m}$  (i.e., carcinoma – a malignant tumor), are actually closer to 46  $\mu\text{m}$  and could have been wrongly classified as a fibroadenoma (benign tumor) if this were a clinical setting. The  $a_{\text{eff}}$  values are all closer to 25  $\mu\text{m}$  than 46  $\mu\text{m}$  avoiding the possibility of a misdiagnosis in this preliminary case. Therefore, our revised algorithm could have a significant impact in the clinic as tissue characterization continues to develop.

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