Diffraction Phenomena with Co-Axial Plane Piston Transducers

A. Goldstein°, D. R. Gandhi*, W. D. O'Brien, Jr.+

°Radiology, WSU, Detroit, MI, *Motorola, Arlington Heights, IL,
+Dept. of ECE, UIUC, Urbana, IL

Abstract

Using an exact theory of Beissner an experimental methodology is developed to accurately measure the effective radii of unfocused, circular plane piston transducers. The transient and steady-state diffraction concepts described by Lord are also used to propose an extension of the definition of the AIUM/NEMA pulse intensity integral to properly analyze cw and tone burst data.

Long standing goals in ultrasound physics have been 1) accurate measurement of unfocused transducer acoustic parameters such as effective radius and 2) accurate experimental verification of steady-state theoretical calculations. Hydrophone measurements use the co-axial geometry shown in Figure 1 to attain these goals. However, diffraction effects due to finite transducer size and propagation geometry must be properly taken into account.

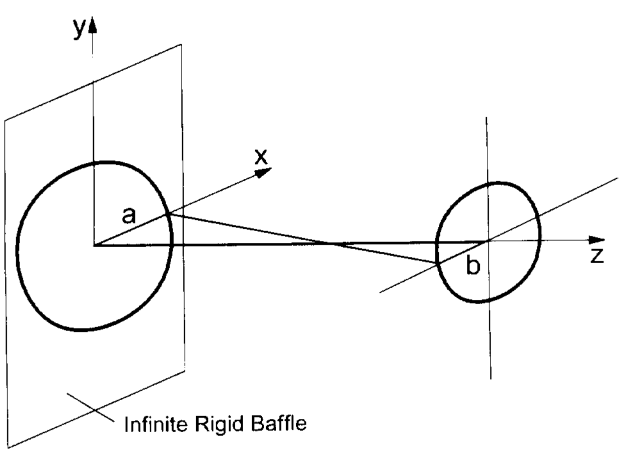


Figure 1. Co-Axial transducer geometry with transmitter of radius a and receiver of radius b. The shortest acoustic path between the two transducers is between their centers and the longest acoustic path is from opposite points on their edges as shown.

The unfocused circular plane piston effective radius can be determined accurately from transducer axial pressure extremum z_m measurements using the known relations [1] for the various maxima and minima

$$a_{eff} = \left(z_m m\lambda + m^2 \frac{\lambda^2}{4} \right)^{1/2} \tag{1}$$

where m = 2n - 1 for maxima, m = 2n for minima, n = 1,2,3... and a_{eff} is calculated for each z_m. The average of all calculated a_{eff} values is taken as the effective radius.

To verify that diffraction caused by a finite hydrophone diameter does not invalidate equation 1 an exact theoretical expression [2] is utilized which gives the diffraction correction D for the case of the two circular co-axial unfocused transducers in Figure 1 with $\gamma = b/a$.

For $\gamma \leq 1$:

$$D = e^{-jkz} - \frac{2}{\pi\gamma^2} \int_{1-\gamma}^{1+\gamma} \sqrt{1 - \left(\frac{1-\gamma^2+\xi^2}{2\xi} \right)^2} \times \exp\left[-jka \sqrt{\xi^2 + \left(\frac{z}{a} \right)^2} \right] d\xi \tag{2}$$

For $\gamma \geq 1$:

$$D = \frac{e^{-jkz}}{\gamma^2} - \frac{2}{\pi} \int_{1-1/\gamma}^{1+1/\gamma} \sqrt{1 - \left(\frac{1-\gamma^{-2}+\xi^2}{2\xi} \right)^2} \times \exp\left[-jka \sqrt{(\gamma\xi)^2 + \left(\frac{z}{a} \right)^2} \right] d\xi \tag{3}$$

Figure 2 shows the unfocused plane piston axial pressure measurement results for a 1 cm radius transducer and several finite dimension hydrophones. Too large a hydrophone causes signal distortion.

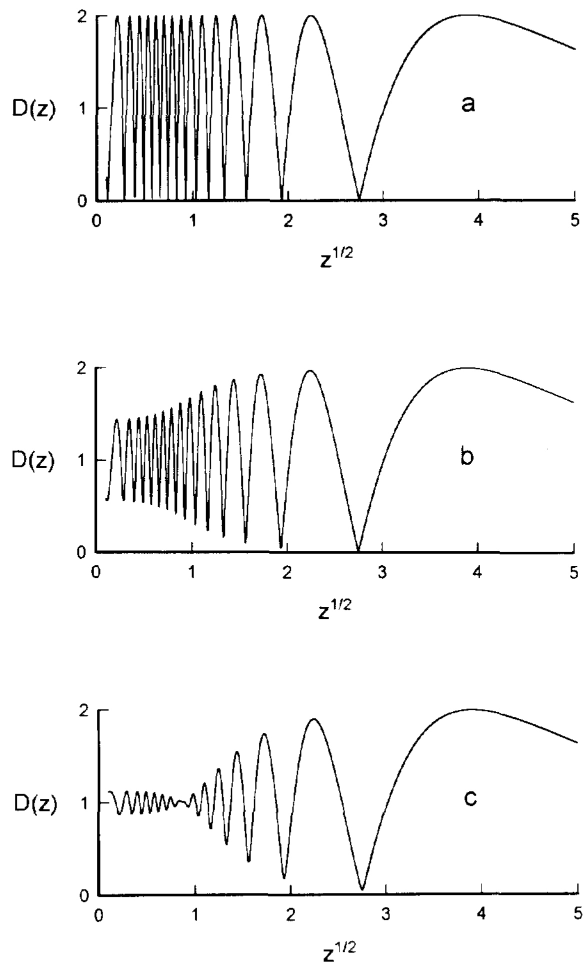


Figure 2. Calculated axial pressure vs. $\gamma^{1/2}$ for a 1 cm radius transducer with several hydrophones: a) 0.02 mm diameter (γ = 0.001), b) 0.5 mm diameter (γ = 0.025) and c) 1 mm diameter (γ = 0.050).

The shift of the steady-state maximum or minimum axial positions with hydrophone radius can be obtained from eqs. 2 and 3 by calculating the magnitude of the diffraction correction (proportional to the received peak signal), adding a water attenuation term [3], differentiating with respect to axial distance and setting the result equal to zero. Using a numerical computer program [4] the fractional change (with respect to extremum values calculated from eq. 1) of the first three axial pressure maxima and minima were obtained (Figure 3). For our experimental case of a ~1 cm radius transmitting transducer and a 0.5 mm diameter receiving hydrophone (γ = 0.025) only the lowest order maxima has an appreciable shift (~1%) in axial position (due to its broadness and the water attenuation).

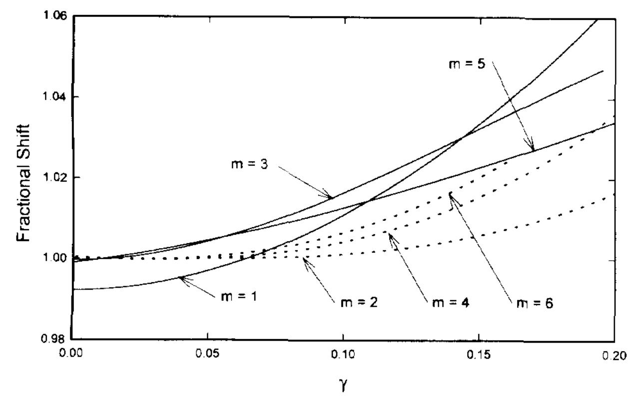


Figure 3. Fractional shift of first six extrema of axial pressure. The fractional shift is defined with respect to the unshifted values of equation 1.

Methods and Materials

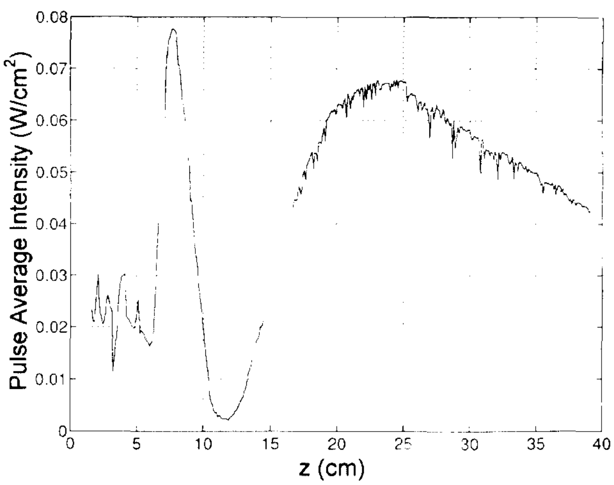
Two unfocused, composite piezoelectric transducers of physical radii 0.938 and 1.25 cm were specially fabricated by Echo Ultrasound for this project. The transducers were suspended in a water tank mounted on a x,y,z Daedal computer controlled precision positioning system at UIUC.

Tone burst transmission was used to simulate steady-state (cw) conditions to avoid the deleterious effects of standing waves and multiple reflections in the tank. A 2.25 MHz, 15 cycle driving tone burst was generated using a Hewlett Packard 8116A signal generator and amplified by an ENI 2100L 50 dB amplifier. The received signal was preamplified by a TEK 11A34 amplifier, displayed on a TEK 11401 digitizing oscilloscope and then peak detected to obtain voltage measurements at 1 mm axial distance increments which were digitized and passed to a Tandy 4000 386 PC for processing. The AIUM/NEMA pulse intensity integral [5] was calculated over the received length of the tone burst in order to obtain the received pulse average intensity (and remove random noise).

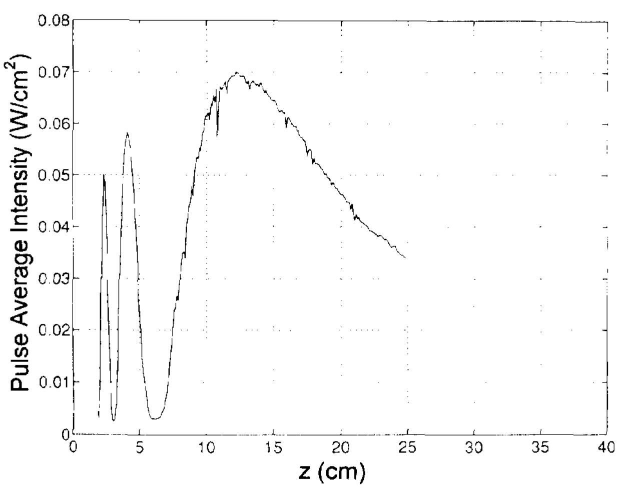
The axial intensity of each transducer was measured using a Marconi 0.5 mm diameter bilaminar hydrophone. Then the two transducers were used in a pitch-catch measurement. Nonlinear propagation effects were avoided by always keeping the transmitter drive voltage low enough so that the second harmonic of the received signal was at least 30 dB below the fundamental.

Results

Figure 4 demonstrates the measured axial intensity at 2.25 MHz for the two Echo Ultrasound transducers. Table 1 presents the measured effective radii, a_{eff}, for the extrema present and the average effective radii for the two transducers. The m = 1 data was corrected using the fractional shift calculated for a 0.5 mm diameter hydrophone measuring the extrema of a 0.90 or 1.25 cm radius transducer.



(a)



(b)

Figure 4. Measured axial pressure at 2.25 MHz: a) 1.27 cm radius transducer and b) 0.95 cm radius transducer.

a_{phys} cm	m	extre-mum	z_m cm	a_{eff} cm	avg a_{eff} cm
0.953	1	max	12.4	0.902	
	2	min	6.05	0.895	
	3	max	4.10	0.901	
	4	min	3.00	0.898	
	5	max	2.30	0.885	0.90
1.27	1	max	24.0	1.26	
	2	min	11.8	1.25	
	3	max	7.75	1.24	1.25

Table 1 Effective radius results from the measured z_m at 2.25 MHz

Figure 5 demonstrates the normalized result of the pitch-catch measurements using the two transducers along with the theoretical prediction for the normalized result corrected for water attenuation.

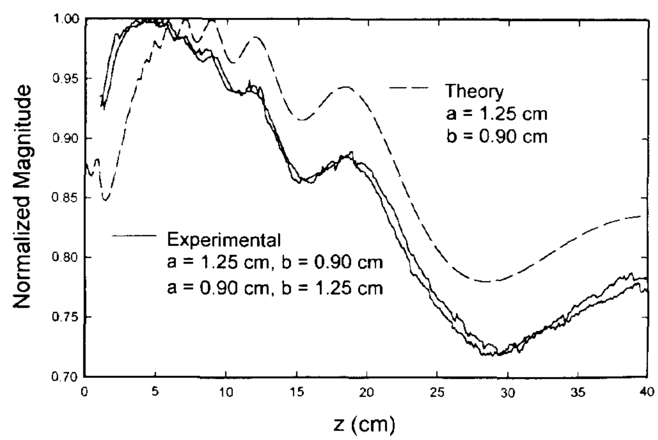


Figure 5. Comparison of theory and experimental results for pitch-catch co-axial measurements using the two Echo Ultrasound transducers.

Discussion

The AIUM/NEMA pulse intensity integral is defined for transmission of short pulses. For cw free field plane waves, when the pulse intensity integral is computed over an integral number of half cycles, the result is the cw intensity. However, in the common experimental case of tone burst transmission, not all of the received tone burst signal is effectively cw steady-state signal.

The received signal is "steady-state" when all portions of the receive transducer front surface in Figure 1 are receiving signal from all portions of the transmit transducer front surface. At the leading and trailing edges of the received tone burst this condition is not satisfied as shown in Figure 6. When the cophasal transmitter emits a tone burst, the face center of the receive transducer is the first region to receive a signal from the transmitter face center (point 1) and the receive face edge is the last region to receive a signal from the opposite transmit face edge (point 2). The time interval between points 1 and 2 is called transient diffraction and was described by Lord [6]. At the trailing edge of the received tone burst, a similar region of transient diffraction occurs between the points 3 (when the receive front face no longer receives signal from the transmit front face) and 4 (when the receive face edge no longer receives signal from the opposite transmit face edge). Between points 2 and 3 exists a region of steady-state diffraction in which the signal amplitude is predictable from steady-state theory. The pulse intensity integral must be calculated over an integral number of half cycles in this "steady-state" diffraction region to obtain the "cw" intensity.

The time duration δ of the transient diffraction region of the received tone burst is equal to the time difference between the face signals being received and the opposing edge signals being received and is given by

$$\delta = \frac{\sqrt{(a+b)^2 + z^2} - z}{c} \tag{4}$$

If the transmitted tone burst contains n cycles with a total length $\tau = n/f$, where f is the transmit frequency, then the total length of the receive tone burst is $\tau + \delta$ (see Figure 6) and the length of the steady-state diffraction region is $\tau - \delta$. The fraction of the receive pulse length which is in steady-state diffraction is $(\tau - \delta)/(\tau + \delta)$. Figure 7 demonstrates this fraction.

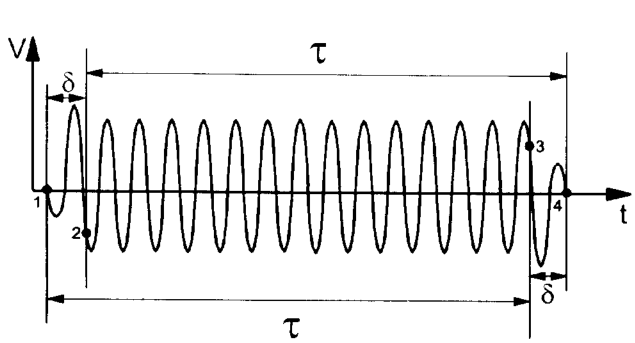


Figure 6. Received tone burst demonstrating regions of transient diffraction at its leading edge (between points 1 and 2) and at its trailing edge (between points 3 and 4).

Figure 7 demonstrates that for the 1.25 cm radius transducer transmitting to the 0.90 cm radius transducer the steady-state region of the received tone burst is substantial only at separations greater than 20 cm. At separations less than 20 cm, the experimental data were compromised resulting in a change of magnitude of the short range signal which affected the normalization and caused the systematic error seen in Figure 5.

It is interesting to note from Figure 7 that even when the axial intensity of each transducer was measured using the 0.5 mm diameter hydrophone, at separations less than 5 cm, there is substantial transient diffraction. This explains the lack of high order extrema in Figure 4. The larger diameter transducer measurement had more transient diffraction and exhibited fewer extrema.

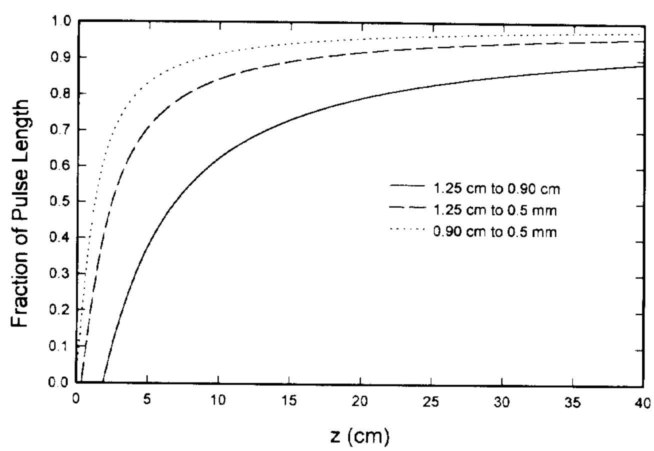


Figure 7. Calculated steady-state portion of the received tone burst length using equation 4 for the 15 cycle transmitted tone burst.

Summary

Theoretical and experimental methodology are presented for using tone burst transmission pulses to measure accurately the effective radii of unfocused, circular plane piston transducers as well as experimentally verify steady-state theoretical calculations. A change in the definition of the AIUM/NEMA pulse intensity integral is proposed to evaluate cw and tone burst experimental data.

References

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