Determination of Absolute Sound Levels and Acoustic Absorption Coefficients by Thermocouple Probes—Theory*

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A detailed theoretical analysis of the operation of thermocouple probes used to determine absolute sound levels or acoustic absorption coefficients is presented.

The probe consists of a thermocouple imbedded in a sound absorbing medium which closely matches in density and sound velocity the medium in which the sound level is to be determined. In use the transducer which generates the acoustic field is excited to generate sound pulses with a rectangular envelope. The initial time rate of change of the temperature at the thermocouple junction is determined. In addition to the measurement of the temperature change, the calculation of the absolute sound intensity requires only a knowledge of the absorption coefficient of the imbedding material and its heat capacity per unit volume at the temperature at which the measurements are made.

The theoretical discussion includes an analysis of the relation between the temperature rise at the junction resulting from absorption in the body of the imbedding medium and the sound level. In addition, the effects of (a) viscous forces arising from relative motion between the thermocouple and the imbedding fluid and (b) heat conduction between the thermocouple and the fluid, in contributing to the temperature change at the thermocouple junction are analyzed.

Based on the analysis, a set of design formulas is obtained which are summarized and illustrated for the convenience of other investigators who may wish to design and use such probes.

I. INTRODUCTION AND TABLE OF SYMBOLS AND DEFINITIONS

THIS paper includes a detailed theoretical analysis of the operation of thermocouple probes used to determine absolute sound levels and acoustic absorption coefficients in liquid media. For absorption coefficient determinations a wire thermocouple is imbedded in the medium and the combination is subjected to a sound pulse of known intensity. The effect of scattering can be eliminated from the measurements by choosing a small sample and imbedding it in a liquid, with a low absorption coefficient, which fairly closely matches it in density and sound velocity. For intensity determinations in liquids the probe consists of a thermocouple and supporting chamber which is filled with a suitable sound absorbing medium. The analysis is concerned with both the temperature change which results from absorption of sound in the interior of the liquid and with the conversion of acoustic energy into heat in the neighborhood of the boundary between the thermocouple and the liquid through viscosity and heat conduction mechanisms.

In discussing the relation between the temperature rise caused by acoustic absorption in the interior of the liquid and that indicated by the thermocouple we are concerned with the effect of (1) the finite heat capacity of the thermocouple wires; (2) heat conduction in the thermocouple wires; (3) the temperature dependence of the absorption coefficient and characteristic impedance of the fluid medium; (4) heat conduction in the fluid; (5) a dependence of acoustic absorption coefficient on space coordinates; and (6) the variation of temperature through the wire.

The analysis of the viscous and heat conduction mechanisms includes (1) calculation of the magnitudes of these effects and (2) estimates of their time course. Cooling of the thermocouple junction, by removal of heat generated at the wire-liquid boundary, resulting from relative motion between the wire and imbedding medium is not discussed since it is assumed that the particle amplitude is small compared to the diameter of the wire. No treatment of scattering is included since: (1) It is assumed that the thermocouple wire diameter is chosen sufficiently small that the effect of scattering from the wire on the temperature distribution can be neglected. A diameter equal to or less than 1/20 of a wavelength is sufficiently small for the probes used in our experimental studies reported in the accompanying paper. (2) For intensity measurements, differences in density and acoustic velocity between the absorbing fluid in which the thermocouple is imbedded and the liquid in which the intensity is to be measured do not impose a basic limitation on the method since these properties could be accurately matched by suitably choosing the absorbing material. (3) In practice, for intensity measurements, the effect of scattering can in many instances be rendered negligible by first choosing an absorbing liquid which matches reasonably closely in density and acoustic velocity the liquid in which the sound field exists and second in choosing an appropriate geometry for the probe. For example, a design such that the thermocouple is imbedded in a thin disk of the absorbing liquid has been convenient for much of our measurements. The disk is oriented so that the direction of propagation of the sound is normal to the plane of the disk.

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Symbols and Definitions

Symbols whose use is restricted to only one section of the paper are not included in this list.

- B defined by expression (98).
- C heat capacity of the imbedding medium per unit mass at constant pressure.
- C_p heat capacity of the imbedding medium per mole at constant pressure.
- C_v heat capacity of the imbedding medium per mole at constant volume.
- C_w heat capacity per unit mass of the wire at constant pressure.
- C_w' $=\rho_w C_w$.
- $f(x), f(r)$ functions describing the variation of the acoustic intensity of the beam of radiation. The coordinate x indicates variation in the direction of the wire. The symbol r designates the radial coordinate measured from the beam center. The intensity is given by $I_0 f(x)$, $I_0 f(r)$.
- G $=kM'$ or $=2hM'$.
- h defined by expression (66) and tabulated in Table II. See also expressions (69a) and (70a).
- h' defined by expression (66) and tabulated in Table II. See also expressions (69b) and (70b).
- I acoustic intensity.
- k see Table I and expressions (53a) and (56a).
- k' see Table I and expressions (53b) and (56b).
- $\mathfrak{k}$ $=K/\rho C$.
- K heat conductivity coefficient of the absorbing medium.
- K_w heat conductivity coefficient of the wire.
- log natural logarithm
- m defined by expression (90).
- m_e $=M'+G$.
- M mass per unit length of wire.
- M' mass of fluid displaced by unit length of wire.
- M_e $=M+G$.
- p acoustic pressure.
- Q_v heat generated per second per unit length of wire by the viscous forces.
- r radial coordinate.
- r_0 radius of the wire.
- R $=k'\omega M'$ or $=2h'\omega M'$.
- $\dot{s}$ velocity of the wire in the direction of propagation of the sound.
- t_e time required to practically realize δT_e.
- δT difference between the temperature of the absorbing medium (linear phase thermocouple absent) and the temperature of the thermocouple junction resulting *only* from heat conduction away from the junction by the wires.
- δT_c difference between the temperature rise of the absorbing medium for $\mu/\rho C$ dependent on the temperature and ΔT_c.
- δT_d difference between ΔT_d and the temperature rise of the absorbing medium with heat conduction present.
- δT_e "equilibrium" difference between the temperature of the absorbing medium, during the "linear" phase of the temperature rise, (thermocouple absent) and the temperature of the thermocouple junction imbedded in the medium under the same conditions of irradiation. δT_e represents only the difference resulting from the finite heat capacity of the thermocouple wires.
- ΔT_c temperature rise of the absorbing medium, $\mu/\rho C$ independent of temperature.
- ΔT_d temperature rise in the absorbing medium neglecting heat conduction in the medium.
- ΔT_m temperature rise in the absorbing medium in the absence of a thermocouple junction.
- $(dT/dt)_0$ time rate of change of the temperature of the imbedding medium at the time of initiation of an acoustic disturbance (rectangular envelope).
- U_{00} particle velocity amplitude of fluid.
- V acoustic velocity in the imbedding medium.
- $\tilde{V}$ molar volume of the imbedding medium.
- W time rate of conversion of acoustic energy into heat per unit length of wire resulting from the mechanism of heat conduction across the boundary.
- x coordinate distance along thermocouple wires.
- γ_0 the ratio of the equilibrium temperature rise at the thermocouple junction resulting from the action of the viscous forces and the temperature rise resulting from acoustic absorption in the body of the imbedding medium in time t.
- ϵ a constant which is numerically of the order of two.
- ζ compressibility of the imbedding medium.
- η coefficient of shear viscosity.
- μ acoustic intensity absorption coefficient per unit path length (imbedding medium).
- ν $=\eta/\rho$.
- $\dot{\xi}$ particle velocity of the fluid.
- ρ density of the imbedding medium.
- ρ_w density of the wire.
- τ coefficient of thermal expansion (volume) of the imbedding medium.
- τ_0 period of acoustic disturbance.
- φ $=(r_0/2)(\omega/\nu)^{\frac{1}{2}}$.
- ψ $=r_0(\omega/\nu)^{\frac{1}{2}}$.
- ω $=2\pi f$.

II. THEORETICAL ANALYSIS

A. Temperature Change Resulting from Absorption in the Liquid

When a region of a fluid medium having an acoustic intensity absorption coefficient μ per unit path length is suddenly subjected to a beam of acoustic radiation of intensity I the initial time rate of change of the temperature, $(dT/dt)_0$, is given by

$$(dT/dt)_0 = \mu I/\rho C. \qquad (1)$$

This relation is a consequence of the fact that at the time of initiation of the acoustic disturbance in the medium the conduction process does not enter the picture. If the absorption coefficient, μ, and the quantity ρC for the medium are known, relation (1) can be used to determine the absolute sound intensity at any given location provided the initial time rate of change of temperature can be measured at that location. In the absence of any temperature measuring device placed at the point, the temperature rises linearly at first as indicated by (1). The time interval during which the rise is closely linear is determined by: the uniformity and width of the beam of radiation, the magnitude of the heat conductivity coefficient for the medium, the rapidity of variation in homogeneity of μ as one moves away from the point, and the temperature coefficients of μ and ρC. As the temperature rises the heat conduction process becomes of increasing importance and equilibrium is approached.

We are interested in determining the temperature as a function of time during the initial linear phase. This can be done by means of a thermocouple. However, the insertion of the thermocouple in the field gives rise to a number of complications which require analysis if we are to obtain, from temperature measurements made with it, accurate values for the temperature rise with the thermocouple absent and values for the initial time rate of change of the temperature with the thermocouple absent. One of these complications is the heating in the region of the boundary between the thermocouple and the imbedding medium which occurs because of (1) the action of viscous forces resulting from relative motion between the thermocouple wires and the fluid medium, (2) a thermal conductivity mechanism involving flow of heat between the wire and the fluid. These mechanisms will be analyzed in the following section. In this section we are concerned first with the difficulties which result because the thermocouple wires have a finite heat capacity and because heat is conducted to or away from the junction by the wires. We assume throughout the analysis in this paper that the contact between the fluid and the wire is such that at the surface of separation the temperatures in the two media are the same, i.e., there is no contact resistance.

1. Effect of the Finite Heat Capacity of the Thermocouple Wires

In the following analysis we assume that the diameter of the thermocouple wire is small compared to a wavelength of the acoustic disturbance.

In the absence of the thermocouple the first or linear phase of the temperature change, ΔT_m in the medium, after initiation of the acoustic disturbance, satisfies the following relation

$$\Delta T_m = (\mu I/\rho C)t. \qquad (2)$$

This follows directly from (1). When the thermocouple is present the temperature rise, resulting from absorption only, experienced by the thermocouple is less than that of expression (2). Heat conduction in the wires is neglected. Let ΔT_{mc} designate the rise in the temperature of the medium at the wire under this condition (with the thermocouple present). Let

$$\delta T = \Delta T_m - \Delta T_{mc}. \qquad (3)$$

The quantity δT is zero at initiation of the disturbance. It subsequently increases at any fixed point in the medium and asymptotically approaches a value independent of time as the time of irradiation increases. For sufficiently small wire diameters this time independent function will be practically realized during the early part of the initial linear phase of the temperature change. Criteria to insure this will be obtained from the analysis in this and subsequent sections.

Since we require only an estimate of the magnitude of δT at the junction an approximate method of calculation will suffice. Consider the system illustrated in Fig. 1. The radius of the wire is designated r_0. Let the

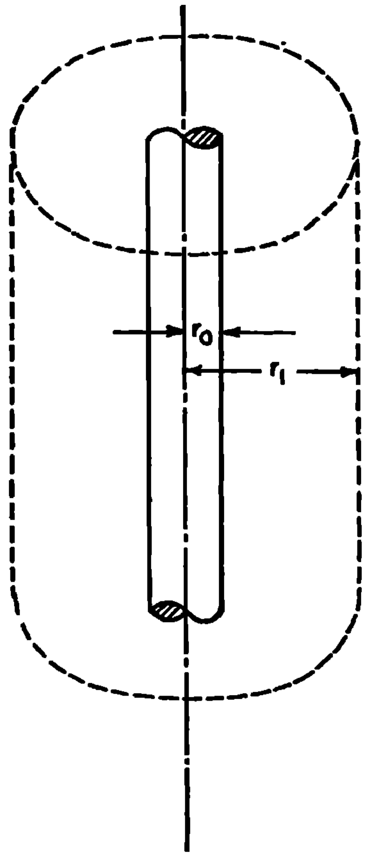

FIG. 1. Thermocouple wire of radius r_0 surrounded by a cylinder of imbedding medium of radius r_1.

intensity of the beam be constant to a distance greater than r_1 from the axis of the wire. Choose the radius r_1 so that the heat capacity per unit length of the medium within the cylinder of radius r_1 is much greater than the heat capacity of a unit length of the wire of radius r_0. If the medium is castor oil (specific heat 0.48) and the wire is either copper (specific heat 0.092) or constantan (specific heat 0.098) a value of 20 for the quantity r_1/r_0 yields a ratio of heat capacities greater than 2000. The temperature change at the distance r_1 can then be taken equal to the right hand side of relation (2). When the expression for ΔT_{mc} obtained from (3) is substituted into the heat flow equation

$$K\nabla^2(\Delta T_{mc})+\mu I=\rho C\frac{\partial(\Delta T_{mc})}{\partial t} \tag{4}$$

we obtain

$$\nabla^2(\delta T)=\left(\frac{\rho C}{K}\right)\frac{\partial(\delta T)}{\partial t}, \tag{5}$$

where K is the coefficient of thermal conductivity for the fluid medium. As indicated above δT ultimately becomes independent of the time. When this situation is realized let $\delta T=\delta T_e$. Then δT_e satisfies the differential equation

$$\frac{d}{dr}\left(r\frac{d(\delta T_e)}{dr}\right)=0. \tag{6}$$

We are assuming that there is negligible variation of temperature along the length of the wire in the neighborhood of the junction. The solution of (6) is

$$\delta T_e=D\log r+E. \tag{7}$$

At $r=r_1$, $\delta T_e=0$ therefore, $E=-D\log r_1$ and (7) can be written

$$\delta T_e=D\log(r/r_1). \tag{8}$$

At $r=r_0$ the time rate of flow of heat into the wire per unit length is $2\pi r_0K(\partial\Delta T_{mc}/\partial r)_{r_0}$ which is equal to

$$-2\pi KD. \tag{9}$$

Since the coefficients of heat conductivity of the metal thermocouple wires are in general much higher than the value of this quantity for the fluid, we can assume that the temperature in the wires is independent of the coordinate r. (See the discussion of this at the end of this section.) In the absence of heat flow along the length of the wire the heat which enters the wire per second per unit length can also be expressed in terms of the time rate of change of temperature of the wire as $\pi r_0^2\rho_wC_w(\partial\Delta T_{mc}/\partial t)$ which is equal to

$$\pi r_0^2\left(\frac{\rho_wC_w}{\rho C}\right)\mu I. \tag{10}$$

Expressions (9) and (10) yield D. The value of δT_e at $r=r_0$ can then be evaluated from (8). The following relation results

$$(\delta T_e)_{r_0}=\frac{\mu I}{2K}r_0^2\left(\frac{\rho_wC_w}{\rho C}\right)\log(r_1/r_0). \tag{11}$$

As a specific example choose $r_1/r_0=20$ as indicated above. Let the remaining quantities assume the following values. The values given for μ, K, and ρC are characteristic of castor oil at about 25°C. The values for ρ_wC_w and K_w are for copper.

I	25	watts/cm²
ρ_wC_w	3.4	joules/cm³/C°
ρC	2.0	joules/cm³/C°
μ	0.12	1/cm (frequency of one megacycle)
K	0.0018	watts/cm/C°
K_w	4.6	watts/cm/C°.

For a wire diameter of 0.003 in. (0.0076 cm) the value of $(\delta T_e)_{r_0}$ is calculated as 0.062 C°. A wire diameter of 0.0005 in. (0.0013 cm) yields a value of $(\delta T_e)_{r_0}$ of 0.0018 C°.

The percentage difference between the temperature at the thermocouple junction and the temperature in the medium, under similar conditions, after a period of time sufficient to insure that $(\delta T_e)_{r_0}$ has been practically realized is obtained from the ratio of (11) and (2) as

$$100\frac{(\delta T_e)_{r_0}}{\Delta T_m}=\frac{100}{t}\left(\frac{\rho_wC_w}{2K}\right)r_0^2\log(r_1/r_0). \tag{12}$$

The ratio r_1/r_0 can be taken equal to 20.

A rough estimate of the time necessary after initiation of the acoustic disturbance for practical realization of the temperature difference $(\delta T_e)_{r_0}$ is required. We first observe that the time rate of change of the temperature of the thermocouple is zero at the instant of initiation of the disturbance. This follows from the observation that the initial temperature gradient between thermocouple and fluid is zero. The situation is illustrated in Fig. 2 for the values of the parameters given above when a copper wire of diameter 0.003 in. is used.

An exact solution for the temperature distribution from the time of initiation of the acoustic disturbance to a time when δT closely approaches δT_e would be quite complex and would not readily yield a simple formula for estimating the time for close approach of δT to δT_e. Accordingly, we will forego any attempt at a precise solution and will use a multiple of the intercept value, on the time axis, of the line $(\Delta T_{mc})_{r_0}=\Delta T_m-\delta T_e$ as an appropriate formula for estimating the time, t_e, required for δT to practically reach the value δT_e.

Symbolically

$$t_e=\epsilon\left(\frac{\rho_wC_w}{2K}\right)r_0^2\log(r_1/r_0) \tag{13}$$

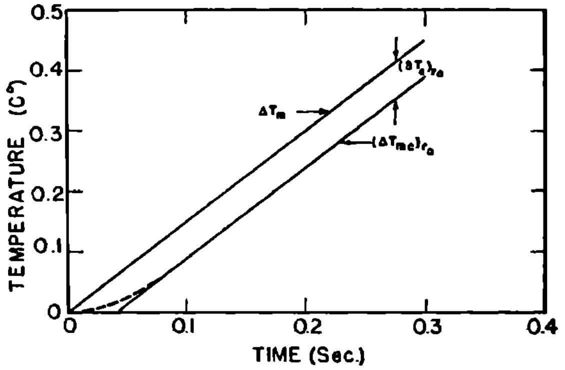

FIG. 2. The initial phase of the temperature rise as a function of the time in an absorbing medium in response to a pulse of sound (a) thermocouple absent (ΔT_m), (b) recorded at thermocouple junction imbedded in the medium, $(\Delta T_{mc})_{r_0}$.

where ϵ is of the order of two as suggested by the graph of Fig. 2. It should be noted that t_e is independent of the sound intensity and the acoustic absorption coefficient.

For the specific example given above with the 0.003 in. diameter copper wire the value of t_e is of the order of 0.08 second. For 0.0005 in. diameter copper wire the time is of the order of 0.002 second. These numerical results are not inconsistent with experimental measurements.

2. *Effect of Heat Conduction in the Thermocouple Wires*

We proceed now with an analysis of the effect of conduction of heat away from the junction by the thermocouple wires on the relation between the temperature of the junction and the temperature of the medium with thermocouple absent under similar conditions of acoustic irradiation. Since we require only an estimate of the magnitude of the effect we base a calculation on the following assumptions.

(1) We choose the radius r_1, at which distance we let the temperature be described as before by relation (2), such that the heat conducted away by the thermocouple wires during the time interval of acoustic irradiation will lower the temperature of a cylinder of the fluid of radius r_1 and length L by an amount which is small compared to the temperature rise produced in the same cylinder of fluid by absorption of the sound. In order to develop a basis for specifying r_1 we first obtain an approximate expression for the heat conducted away by the wires. Consider the situation illustrated in Fig. 3. The thermocouple junction is at position x_0. For the purpose of this evaluation we can assume that the temperature distribution along the wire, in the x direction, is the same as if the wire were not present (this follows because we are only interested in situations in which the percentage difference between the temperature at the wire and the temperature in the medium with the thermocouple absent is small). The temperature distribution to be used in this evaluation can then be obtained by using relation (2) in conjunction with the intensity beam pattern of the irradiator. The length L is small enough that the intensity does not change by a large factor in a distance L. For wires with equal heat conductivity coefficients, the heat, dH/dt, conducted away per unit time by the wires from the length L is given approximately by the expression

$$\frac{dH}{dt} = -\pi r_0^2 K_w \left(\frac{\partial^2 \Delta T_m}{\partial x^2}\right)_{x_0} L. \tag{14}$$

If the wires have different heat conductivity coefficients then the heat, H, conducted away from the length L per second by the wires is expressed approximately by

$$\frac{dH}{dt} = \pi r_0^2 \left[\left(K_{w1}\frac{\partial \Delta T_m}{\partial x}\right)_{x_0 - L/2} - \left(K_{w2}\frac{\partial \Delta T_m}{\partial x}\right)_{x_0 + L/2}\right]. \tag{15}$$

The total amount of heat, H_T, removed from the region by conduction through the wires during the period of irradiation is given by the integral

$$H_T = \int_0^\tau \frac{dH}{dt} dt \tag{16}$$

where τ is the period of irradiation. It is convenient to express the intensity beam pattern in the form

$$I_0 f(x) \tag{17}$$

where x designates the coordinate distance along the direction of the wire, $f(0)=1$, and $x=0$ designates the position of peak intensity. The temperature function to be inserted into (14) and (15) is then obtained, as indicated above, by combining (2) and (17)

$$\Delta T_m = (\mu/\rho C) I_0 f(x) t. \tag{18}$$

Consider a specific example. Choose the value of the peak intensity I_0 so that $\partial \Delta T_m / \partial t = 1.5$C°/sec., i.e., $\Delta T_m = 1.5 f(x) t$. Let the thermocouple be placed at the peak $x_0 = 0$. The beam pattern for the focusing irradiator used in the studies reported in this paper is shown in Fig. 4. This pattern was taken transverse to the axis

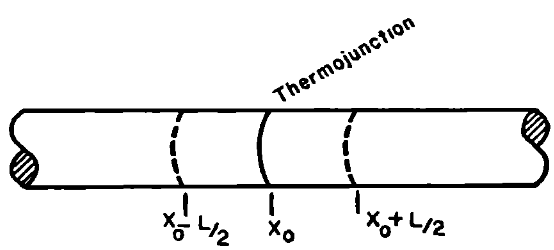

FIG. 3. An element of the thermocouple wire. Coordinates used in the analysis of the effect of heat conduction along the length of the wire. The junction is positioned at x_0.

of the beam in the focal region. A parabolic curve fitting near the peak yields $f(x)=1-1.48x^2$. The temperature function then has the form

$$\Delta T_m=1.5(1-1.48x^2)t. \quad (19)$$

If one wire of the thermocouple is copper and the other constantan we combine (15), (16), and (19) and obtain (the heat conducted by the constantan can be neglected since the thermal conductivity coefficient of copper is twenty times that of constantan)

$$H_T=\int_0^\tau \frac{dH}{dt}dt=1.11\pi r_0^2 K_{wc} L\tau^2, \quad (20)$$

where K_{wc} is the thermal conductivity coefficient of copper ($K_{wc}=4.6$ watts/cm/C°). For a wire diameter of 0.003 in. (0.0076 cm), $L/2=20r_0$ and $\tau=1.0$ sec the value of H_T is $3.5(10)^{-5}$ joule. The energy which is imparted to the fluid medium (castor oil) by absorption of acoustic energy in the cylinder of radius r_1 (choose $r_1=20r_0$) and length L in one second is equal to the product of the time rate of change of the temperature, the heat capacity per unit volume and the volume. This is $8.3(10)^{-3}$ joule. The heat absorbed by the fluid medium is thus over 200 times that conducted away by the thermocouple wires.

(2) We assume that the gradient of the temperature in the direction of the wire (x axis) at $r=r_0$ is small enough and that the temperature distribution adjusts quickly enough so that a function of r of the form used in the analysis of the effect of the finite heat capacity of the wire can describe approximately the temperature distribution between r_0 and r_1 at each instant of time. Refer to expressions (3) and (8).

Consider first the variation of temperature in the direction of the wire. We are interested in the fractional change, δ, in the temperature along a length, $L/2$, of wire where the magnitude of $L/2$ is of the order of r_1, say $L/2=r_1$. The fractional change in temperature is given by the expression

$$\delta=\frac{|\partial\Delta T_m/\partial x|}{\Delta T_m}(L/2) \quad (21)$$

where ΔT_m is given by (18). Combining (18) and (21) we obtain for the fractional change in the temperature along the length $L/2$ of wire

$$\delta=(L/2)\left|\frac{df(x)}{dx}\right| \Big/ f(x). \quad (22)$$

Let $L/2=20\,r_0$ as before. For a wire diameter of 0.003 in. and with the thermocouple positioned at the peak of the beam pattern, Fig. 4, we obtain $\delta=0.017$. In the calculation, the maximum value of $|df/dx|$ in the interval $0\leq x\leq L/2$, i.e., $|df/dx|_{L/2}$ was used. For a wire diameter of 0.0005 in. we obtain $\delta=0.00047$. These values make our assumption regarding the form of the temperature distribution function reasonable if the second requirement, that the time rate of change of the temperature is sufficiently slow, is satisfied.

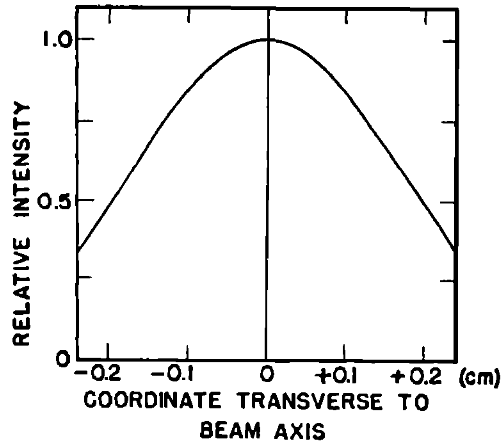

FIG. 4. Beam pattern in the focal region transverse to the direction of propagation.

We consider next the assumption that the temperature distribution adjusts quickly enough so that an equilibrium distribution is a reasonable approximation to this distribution. We first estimate the temperature difference, between the thermocouple junction and the medium with the thermocouple absent, caused by heat conduction in the thermocouple wires.

Refer to Fig. 5. In line with the previous discussion, we take for the temperature distribution function ΔT_{mc} in the space between r_0 and r_1 the form (combining (3) and (8))

$$\Delta T_{mc}=\frac{\mu I}{\rho C}t-D\log(r/r_1). \quad (23)$$

The quantity D varies only slowly with x as discussed above; it is also a function of the time. Consider the element of wire between x_1 and $x_1+\Delta x$. We assume that the temperature is uniform across the wire. In order that the temperature of this element of the wire remain constant the heat transferred per unit time to the wire from the fluid medium must just balance the heat conducted away per second from the element by the wire. When this is expressed symbolically we have

$$\pi r_0^2 K_w\left[-\left(\frac{\partial\Delta T}{\partial x}\right)_{x_1}+\left(\frac{\partial\Delta T}{\partial x}\right)_{x_1+\Delta x_1}\right] +2\pi r_0\Delta x K\left(\frac{\partial\Delta T_{mc}}{\partial r}\right)_{r_0}=0 \quad (24)$$

where ΔT is the increase in temperature of the wire over its initial value. Upon taking the limit as $\Delta x\to 0$ and combining with (23) we evaluate D

$$D=\frac{K_w r_0^2}{2K}\frac{\partial^2\Delta T}{\partial x^2}. \quad (25)$$

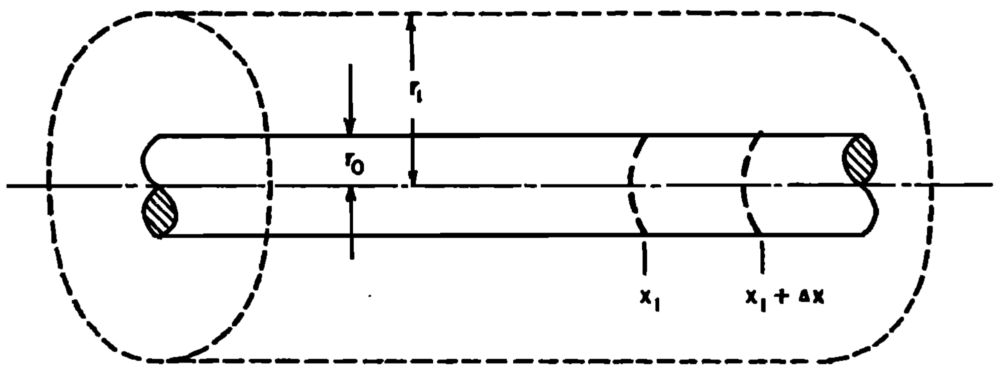

FIG. 5. An element of the thermocouple wire. Coordinates for analyzing the effect of heat conduction.

The difference in temperature, δT, between the wire and the medium without the wire present is then

$$\delta T = \left[\frac{\mu I}{\rho C} t - \Delta T_{mc}\right] = -\frac{K_w r_0^2}{2K}\log(r_1/r_0)\frac{\partial^2 \Delta T}{\partial x^2}. \quad (26)$$

In order to evaluate numerically δT from (26) we can take for ΔT, as we did in the previous analysis, a temperature distribution function along the wire corresponding to the intensity distribution function of the acoustic disturbance. The magnitude of $\partial^2 T/\partial x^2$ for this distribution will not be smaller than that for the actual distribution since conduction in the wires acts to decrease the temperature gradients. We let (refer to (18))

$$\Delta T = \Delta T_m = \frac{\mu}{\rho C} I_0 f(x) t. \quad (27)$$

We compute the ratio of δT to ΔT,

$$\frac{\delta T}{\Delta T} = -\frac{K_w r_0^2}{2K}\log(r_1/r_0)\frac{f''(x)}{f(x)}. \quad (28)$$

As a specific example we consider a copper wire of diameter 0.003 in., castor oil as the fluid medium and the beam pattern introduced previously. The ratio r_1/r_0 is taken equal to 20. Then $\delta T/\Delta T = 0.17$ at the peak of the beam. For a copper-constantan thermocouple, 0.003 in. wire diameter, in this position, the value of $\delta T/\Delta T$ is about one-half of the result just obtained since the thermal conductivity coefficient of constantan is about 1/20 that of copper, i.e. $\delta T/\Delta T = 0.09$. When the copper wire diameter is 0.0005 in. the computed value of $\delta T/\Delta T$ from formula (26) is 0.0046.

We consider now the limitation imposed on the system by the requirement, made for the purpose of simplifying the analysis, that an equilibrium distribution can be used to approximately describe the temperature distribution between r_0 and r_1 at each instant of time. We note first that at the time of initiation of the acoustic disturbance the temperature difference, δT, between surfaces r_1 and r_0, is zero. After initiation of the disturbance it increases monotonically. A rough estimate of the time, t_e, required for the temperature distribution function, which describes the difference in temperature of the medium between r_1 and r_0 with the thermocouple present and the temperature with the thermocouple absent, to practically attain its equilibrium form for any specified temperature difference, δT_e, between the wire and r_1 can be obtained from relations (11) and (13). If we combine the two relations we obtain

$$t_e = \delta T_e \frac{\epsilon}{(\mu I/\rho C)} \quad (29)$$

where ϵ is numerically of the order of two. Now (29) was derived from relations obtained in the analysis of the effects of the finite heat capacity of the wire neglecting conduction. The value of t_e for the physical situation is, therefore, probably somewhat less than the value computed from (29) since conduction in the wire would tend to bring about an "equilibrium" distribution more quickly than in the case of no conduction for equal temperature differences between the wire and the surface $r = r_1$. If we insert into (29) for δT_e the value of δT from (26) and substitute for ΔT the right hand side of expression (27) we obtain an expression for the ratio of the time required for practical equilibrium, t_e, and the time of irradiation, t,

$$\frac{t_e}{t} = -\frac{K_w r_0^2}{2K}\log(r_1/r_0)\frac{f''(x)}{f(x)}\epsilon. \quad (30)$$

This expression is identical in form with expression (28) with the exception of the factor ϵ. If the quantity t_e/t is a small fraction, say of the order of $\frac{1}{5}$ or less, then it is considered that the use of the equilibrium distribution function is reasonable. We obtain now numerical estimates of t_e/t for specific cases. Refer to the numerical results following expression (28). For 0.003 in. diameter copper placed at the peak of the beam pattern with castor oil as the medium, $t_e/t = 0.33$. For a copper-constantan junction at the peak $t_e/t = 0.17$. For 0.0005 in. diameter copper wire at the peak $t_e/t = 0.0092$. It is clear that a 0.0005 in. diameter copper-constantan thermocouple in castor oil used in conjunction with a focusing irradiator having a beam pattern approximating that of Fig. 4 easily satisfies the above criterion.

From the above analysis of the effect of the finite heat conductivity of the thermocouple wires and the conduction of heat by the wires it follows, to the extent that these two mechanisms are involved, that an acoustic probe consisting of a 0.0005 in. diameter copper-constantan thermocouple in castor oil is entirely satisfactory to use with a focusing irradiator having a beam pattern as narrow as that illustrated in Fig. 4.

We are concerned next with the factors which, in the absence of the thermocouple, ultimately cause a deviation in the temperature time functions from the linear form quantitatively described by (2).

3. *Effect of the Temperature Dependence of the Quantity $\mu/\rho C$ on the Initial Phase of the Temperature Change*

We are interested in the deviation of the temperature-time functional relation from the linear form, described by expression (2), caused by the dependence of the quantity $\mu/\rho C$ on the temperature. Since we are interested in the initial phase of the deviation from linearity, it is sufficient to use only the first two terms in the power series expansion of the quantity $\mu/\rho C$ as a function of the temperature, i.e.,

$$\mu/\rho C=(\mu/\rho C)_0+(\partial[\mu/\rho C]/\partial T)_0\Delta T_m \qquad (31)$$

where ΔT_m is the temperature change in the medium resulting from absorption of acoustic energy. The subscript zero indicates evaluation of the subscripted quantity at the temperature of the medium before absorption of the sound. In place of relation (1) we have the following:

$$\frac{d\Delta T_m}{dt}=[(\mu/\rho C)_0-\zeta_0\Delta T_m]I \qquad (32)$$

where

$$\zeta_0=-(\partial[\mu/\rho C]/\partial T)_0.$$

Now at $t=0$, $\Delta T_m=0$ so that the appropriate solution of (32) is

$$\Delta T_m=(\mu/\rho C)_0[1-e^{-\zeta_0 I t}]/\zeta_0. \qquad (33)$$

Restrict the values of t under consideration so that $\zeta_0 It\ll 1$. Then (33) can be written in the approximate form

$$\Delta T_m=(\mu/\rho C)_0 It[1-\zeta_0 I(t/2)]. \qquad (34)$$

The term $\zeta_0 I(t/2)$ is a measure of the deviation from the linear form described by expression (2). For convenience in applying corrections to experimental data either for the evaluation of acoustic absorption coefficients or for determination of absolute sound levels this quantity can be written in the following approximate form:

$$\zeta I(t/2)\simeq[\zeta_0/2(\mu/\rho C)_0]\Delta T_m. \qquad (35)$$

As a specific example the value of $\zeta_0/2(\mu/\rho C)_0$ is about 0.04 for castor oil at 25°C. Thus for a 1°C temperature rise the deviation of the temperature time function from a linear relation will amount to about 4 percent.

4. *Effect of Heat Conduction in the Fluid on the Initial Phase of the Temperature Change*

We consider now the deviation of the temperature-time relation from the linear form, described by expression (2), caused by heat conduction in the fluid with the thermocouple absent. Since we are primarily interested in the magnitude of this effect for the purpose of evaluating the uncertainty in the determination of absorption coefficients or absolute sound levels, we will restrict the discussion here to the effect in the neighborhood of the peak of the beam of radiation. The extension to other locations in the beam is readily accomplished. Consider a circular cylinder of radius Δr in the fluid whose axis is oriented in the direction of propagation of the sound, and coincides with the intensity maximum of the beam. (See Fig. 6.)

The fractional deviation of the temperature at the peak of the beam, with heat conduction present, from the value of the temperature with heat conduction neglected (described by relation (2)) is equal to the limit of the ratio of the heat conducted away from the cylinder per unit length and the acoustic energy absorbed in the cylinder as $\Delta r\to 0$. Since we are interested in situations in which this ratio is small, it is suitable for the purpose of evaluating the heat conducted away

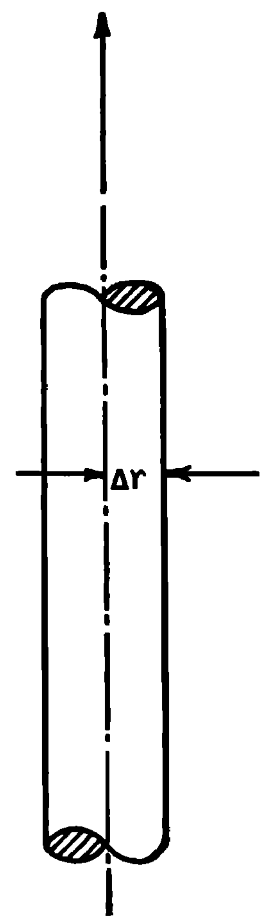

FIG. 6. A circular cylindrical element of an absorbing medium.

from the cylindrical element to use the temperature distribution which would be produced by the beam in the fluid medium if conduction is neglected. Let ΔT represent this temperature distribution. It is a function of the coordinate r and the time. We assume that the beam pattern has cylindrical symmetry. Then the heat H_c conducted away from the cylinder per unit length per unit time is $-2\pi\Delta r K(\partial\Delta T/\partial r)_{\Delta r}$. The partial derivative is expanded about $r=0$. Since the beam maximum is positioned at $r=0$, $(\partial\Delta T/\partial r)_{r=0}=0$ and

$$H_c=-2\pi K(\Delta r)^2(\partial^2\Delta T/\partial r^2)_{r=0}. \quad (36)$$

The heat H_a absorbed in the cylinder per unit length per unit time is

$$H_a=\pi(\Delta r)^2\rho C(\partial\Delta T/\partial t)_{r=0}. \quad (37)$$

Let the intensity beam pattern be represented by $I_0 f(r)$ where $f(0)=1$. Then the temperature distribution function ΔT can be expressed in terms of this beam pattern function as was done previously:

$$\Delta T=(\mu/\rho C)I_0[f(r)]_{r=0}t. \quad (38)$$

The heat H_{ct} conducted away from the cylindrical element in time t is obtained by integration of expression (36) where ΔT has the form given by (38). The following form results

$$H_{ct}=-\pi K(\Delta r)^2(\mu I_0/\rho C)f''(r)t^2. \quad (39)$$

Similarly the heat H_{at} absorbed in the element of volume in time t is evaluated by integrating (37);

$$H_{at}=\pi(\Delta r)^2\mu I_0 f(r)t. \quad (40)$$

The limit of the ratio H_{ct}/H_{at} as $\Delta r\rightarrow 0$ is then

$$\left(\frac{H_{ct}}{H_{at}}\right)_0=-\frac{K}{\rho C}\left(\frac{f''(r)}{f(r)}\right)_{r=0}t. \quad (41)$$

As a specific example consider the situation for the beam pattern of Fig. 4, $f(r)=1-1.48\,r^2$, and for castor oil as the fluid medium. The value of $(H_{ct}/H_{at})_0$ for this case is $0.0027t$. The deviation of the temperature caused by heat conduction in the oil from the value given by relation (2) is, therefore, less than 0.3 percent at the end of a one second period of irradiation.

5. Effect of a Dependence of μ on the Space Coordinates on the Initial Phase of the Temperature Change

This paper is primarily concerned with a study of the temperature changes which are experienced by a thermocouple imbedded in a homogeneous fluid medium which is exposed to an acoustic disturbance. An important objective is the analysis of various factors which would, if not understood with respect to magnitude, result in uncertainties in the values of absolute sound levels calculated from data obtained with a probe operating on the principle of initial time rate of change of temperature. However, if the absolute sound level is known the same data yield a value for the acoustic intensity absorption coefficient for the imbedding fluid. In nonhomogeneous material, such as tissue, it is desirable to determine the absorption coefficient as a function of position in the material. For such materials an estimate of the effect of space variations of μ on the form of the temperature-time function is desirable. This is necessary in order to specify the accuracy with which a given space distribution of μ is determined by this method for a particular experimental arrangement and procedure. The detailed analysis is not included here but it is noted that estimates can be made by letting μ of expression (38) be a function of r. The differentiation indicated in (36) would then include a term involving $(d\mu/dr)$. The limitation on t in order to realize an arbitrarily specified degree of accuracy in the determination of μ at the position of the thermocouple junction would then follow from the ratio $(H_{ct}/H_{at})_0$.

6. Uncertainty in the Thermocouple Junction Temperature Resulting from the Variation of the Temperature Within the Wire with the Radial Coordinate

We are concerned herein with the magnitude of the uncertainty in the temperature of the thermocouple junction resulting from the nonuniformity in the temperature distribution over the cross section of the wire. For the purpose of estimating the magnitude of this effect, we let the temperature change at the interface between the wire and the imbedding medium be given by the relation (2). Since we are only interested in the effect of a variation of the temperature with radius, and not with conduction along the length of the wire which has been analyzed previously, we take $(\mu I/\rho C)$ constant along the wire. At the time of initiation of the irradiation, the temperature difference between the interior of the wire and the boundary is zero. The temperature difference then begins to increase and after a period of time attains a value which is essentially constant in time. We consider the situation after this constant value has been realized. Let the temperature change, ΔT_w, in the wire be written as

$$\Delta T_w=\left(\frac{\mu I}{\rho C}\right)t-\delta T_w, \quad (42)$$

where δT_w is independent of the time. When this expression for ΔT_w is substituted into the equation

$$K_w\nabla^2(\Delta T_w)=\rho_w C_w\partial(\Delta T_w)/\partial t, \quad (43)$$

the following result is obtained:

$$\frac{1}{r}\frac{d}{dr}\left[r\frac{d\delta T_w}{dr}\right]=-\frac{\mu I}{K_w}\left(\frac{\rho_w C_w}{\rho C}\right). \quad (44)$$

The appropriate solution of (44) is

$$\delta T_w=\left(\frac{\rho_w C_w}{\rho C}\right)\frac{\mu I}{4K_w}(r_0^2-r^2). \quad (45)$$

The maximum temperature deviation, δT_{wm}, from the value at the boundary of the wire is obtained at $r=0$

$$\delta T_{wm}=\left(\frac{\rho_w C_w}{\rho C}\right)\frac{\mu I}{4K_w}r_0^2. \quad (46)$$

The ratio, $\delta T_{wm}/\Delta T_m$, of this temperature deviation from the boundary value and the value, $\Delta T_m=(\mu I/\rho C)t$, at the boundary is

$$\frac{\delta T_{wm}}{\Delta T_m}=\left(\frac{\rho_w C_w}{4K_w}\right)\frac{r_0^2}{t}. \quad (47)$$

For constantan wire of 0.003 in. diameter (0.0076 cm) and $t=1.0$ sec this ratio is of the order of $(10)^{-4}$.

B. Temperature Changes Resulting from Conversion of Acoustic Energy into Heat at Wire Boundary

1. Viscosity

The temperature changes produced when the region of a fluid medium in which a wire thermocouple is located is subjected to acoustic radiation result partially from the action of the viscous forces between the wire and the fluid. It is necessary to estimate, at least roughly, both the magnitude of the temperature change resulting from this and the time required for the distribution to reach a specified fraction of its equilibrium value. Since we do not require accurate theoretical values for these quantities, we will consider only the plane wave case. In order to understand the directionality characteristics of such a probe, it is desirable to carry out computations for two orientations of the wire with respect to the direction of propagation of an acoustic disturbance. The two cases considered are: (a) direction of propagation along the direction of the wire, (b) direction of propagation at right angles to the direction of the wire. Any other orientation can be treated by compounding these two in a suitable linear combination.

We will first obtain expressions for the time rate of conversion of acoustic energy into heat at the wire. Since the diameter of the wire is of necessity small compared to one wavelength of the sound, we will utilize expressions for the forces of frictional resistance and inertia experienced by an oscillating incompressible wire in an incompressible viscous medium.

(*a*) *Wire transverse to direction of propagation.*—Consider a wire immersed in a fluid medium in which a plane wave of sound is propagating in a direction at right angles to the axis of the wire. See Fig. 7(a). Let $\dot{\xi}$ represent the particle velocity of the fluid and let $\dot{s}$ represent the velocity of the wire in the direction of propagation of the sound. The following differential equation is used to describe the motion

$$M\ddot{s}=-R(\dot{s}-\dot{\xi})-G(\ddot{s}-\ddot{\xi})+M'\ddot{\xi} \quad (48)$$

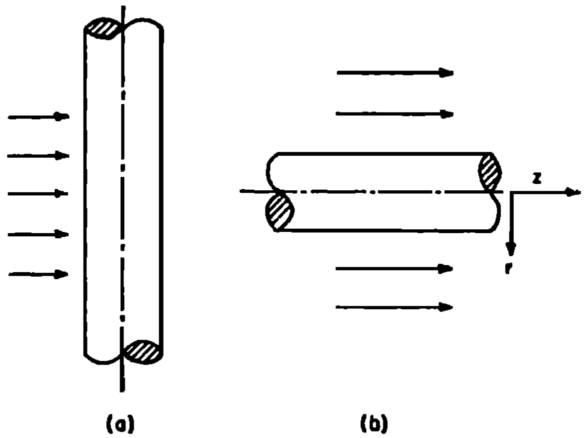

FIG. 7. (a) A wire subjected to a plane acoustic wave propagating in a direction perpendicular to the axis of the wire. (b) A wire immersed in a fluid medium which is flowing in the direction of the axis of the wire.

where M is the mass per unit length of the wire and M' is the mass of fluid displaced by unit length of wire. The first and second terms on the right hand side of the differential equation represent the frictional and inertia forces experienced by an oscillating cylinder in an incompressible fluid medium and the third term represents the driving force of the incident sound wave. The differential equation (48) can be written in the following form[1]

$$\ddot{s}+(R/M_e)\dot{s}=(R/M_e)\dot{\xi}+(m_e/M_e)\ddot{\xi} \quad (49)$$

where

$$M_e=M+G$$
$$m_e=M'+G$$

We will designate M_e the "effective mass of the wire per unit length" and m_e the "effective mass of the fluid element displaced by unit length of the wire." Let $\dot{\xi}=U_{00}e^{i\omega t}$. Under steady state conditions the amount of acoustic energy, Q_v, converted into heat per second per unit length of wire is

$$Q_v=\frac{1}{\tau_0}\int_0^{\tau_0}R(\dot{s}-\dot{\xi})^2dt=\frac{U_{00}^2R}{2}\frac{\omega^2[1-(m_e/M_e)]^2}{\omega^2+(R/M_e)^2}. \quad (50)$$

We now require expressions for the "effective" mass and the frictional constant per unit length of wire. Relations for these quantities were obtained by Stokes for a circular cylinder oscillating in an incompressible viscous medium in a direction perpendicular to the axis of the cylinder.[2] In this investigation the inertia forces were neglected. Following Stokes we write for the effective masses the expressions

$$M_e=M+kM'$$
$$m_e=M'+kM' \quad (51)$$

[1] This equation appears for example in a paper by Angerer, Barth, and Guttner, Strahlentherapie 84, 601 (1951).

[2] G. G. Stokes, Trans. Cambridge Phil. Soc. 9, Part II, 15–62 (1851).

TABLE I.

φ	k	k'	$\varphi^2 k$	$\varphi^2 k'$	φ	k	k'	$\varphi^2 k$	$\varphi^2 k'$
0.0	∞	∞	0.0000	0.0000	2.1	1.677	0.7822	7.395	3.450
0.1	19.70	48.63	0.1970	0.4863	2.2	1.646	0.7421	7.966	3.592
0.2	9.166	16.73	0.3666	0.6691	2.3	1.618	0.7059	8.557	3.734
0.3	6.166	9.258	0.5549	0.8332	2.4	1.592	0.6730	9.168	3.877
0.4	4.771	6.185	0.7633	0.9896	2.5	1.568	0.6430	9.799	4.019
0.5	3.968	4.567	0.9920	1.142	2.6	1.546	0.6154	10.45	4.160
0.6	3.445	3.589	1.240	1.292	2.7	1.526	0.5902	11.12	4.303
0.7	3.082	2.936	1.510	1.439	2.8	1.507	0.5669	11.81	4.444
0.8	2.812	2.477	1.800	1.585	2.9	1.489	0.5453	12.52	4.586
0.9	2.604	2.137	2.110	1.731	3.0	1.473	0.5253	13.25	4.728
1.0	2.439	1.876	2.439	1.876	3.1	1.457	0.5068	14.01	4.870
1.1	2.306	1.678	2.790	2.021	3.2	1.443	0.4895	14.78	5.012
1.2	2.194	1.503	3.160	2.164	3.3	1.430	0.4732	15.57	5.154
1.3	2.102	1.365	3.552	2.307	3.4	1.417	0.4581	16.38	5.296
1.4	2.021	1.250	3.961	2.450	3.5	1.405	0.4439	17.21	5.437
1.5	1.951	1.153	4.389	2.595	3.6	1.394	0.4305	18.06	5.580
1.6	1.891	1.069	4.841	2.739	3.7	1.383	0.4179	18.93	5.721
1.7	1.838	0.9965	5.312	2.880	3.8	1.373	0.4060	19.82	5.863
1.8	1.791	0.9332	5.804	3.024	3.9	1.363	0.3948	20.73	6.005
1.9	1.749	0.8767	6.314	3.165	4.0	1.354	0.3841	21.67	6.145
2.0	1.711	0.8268	6.845	3.307	∞	1.000	0.0000	∞	∞

and for the frictional constant R the expression

$$R = k'M'\omega. \tag{52}$$

The quantities k and k', which are expressed as functions of a parameter φ, are tabulated by Stokes for a range of values of the parameter over which numerical computation is most tedious. The parameter φ is equal to $(r_0/2)(\omega/\nu)^{\frac{1}{2}}$ where r_0 is the radius of the wire and ν is the kinematic coefficient of shear viscosity. For convenience we include Stokes' table of calculated values of k and k' as Table I of this paper. The symbol φ replaces the bold-faced lower case letter m of Stokes' paper.

As $\varphi \to 0$ k and k' are given by the following expressions obtained by Stokes

$$k = 1 + (1/\varphi^2)(\pi/4)/[(\log\varphi + \gamma)^2 + (\pi/4)^2] \tag{53a}$$

$$k' = (1/\varphi^2)[-(\log\varphi + \gamma)]/[(\log\varphi + \gamma)^2 + (\pi/4)^2] \tag{53b}$$

where γ is Euler's constant ($\gamma = 0.577\cdots$).

Now insert for φ in (53b) its explicit form in terms of ω and substitute the resulting expression for k' into (52). We then observe that the frictional constant $R \to 0$ as $\omega \to 0$. It is of interest to compare this result with the following. The resistance per unit length of a circular cylinder moving with a constant velocity of translation, U_1, perpendicular to its axis through a viscous liquid is given accurately by the expression

$$4\pi\eta U_1/[\tfrac{1}{2} - \gamma - \log(r_0 U_1/4\nu)] \tag{54}$$

when the Reynolds number, $2r_0U_1/\nu$, is numerically less than one.[3-5] The quantity η is the coefficient of shear viscosity. The frictional parameter, R', for this case, which corresponds to the frictional constant R of the preceding analysis, is equal to the resistance divided by the velocity. Thus

$$R' = 4\pi\eta/[\tfrac{1}{2} - \gamma - \log(r_0 U_1/4\nu)]. \tag{55}$$

Now the expression obtained by Stokes for the frictional constant for the vibrating cylinder was obtained under the condition that the amplitude of the motion is small compared to the radius of the cylinder. As $\omega \to 0$ this implies that the velocity amplitude approaches zero. We, therefore, consider expression (55) as $U_1 \to 0$. It is readily seen that $R' \to 0$. For values of $\varphi \geq 4$, k and k' are given by the approximate expressions

$$k = 1 + (\sqrt{2}/\varphi) \tag{56a}$$

$$k' = (\sqrt{2}/\varphi) + \tfrac{1}{2}\varphi^2. \tag{56b}$$

The quantity Q_v can be written as follows:

$$Q_v = \frac{U_{00}{}^2 R}{2} \frac{[1 - \{(1+k)/[(M/M')+k]\}]^2}{1 + [k'/[(M/M')+k]]^2}. \tag{57}$$

Before obtaining a formula for estimating the change in temperature of the wire which results from the conversion of acoustic energy into heat as given by expression (57) we will obtain a corresponding formula for the case of acoustic propagation along the axis of the wire.

(*b*) *Wire in direction of propagation.*—Since the diameter of the wire is small compared to one wavelength of the sound, we will calculate the effective masses and frictional resistance for a wire vibrating in the direction of its axis in a viscous incompressible medium with all elements of the wire in the same time phase. We will then apply these results to the acoustic case where the time phase of the relative motion between wire and fluid varies in the direction of the axis of the wire.

Let the direction of the axis of the wire lie along the z coordinate as illustrated in Fig. 7(b). Since the motion in the fluid is caused by the oscillatory motion of the wire, which is considered infinitely long, the only non-zero component of velocity in the fluid is v_z which is a function only of r and t. The equations of motion then reduce to

$$\rho\frac{\partial v_z}{\partial t} = \frac{\eta}{r}\left[\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right)\right] - \frac{\partial p}{\partial z}, \tag{58a}$$

$$\frac{\partial p}{\partial \theta} = 0, \quad \frac{\partial p}{\partial r} = 0. \tag{58b}$$

From (58) we note that $\partial p/\partial z$ must be independent of z. Let

$$\begin{aligned} v_z &= Ue^{j\omega t} \\ P &= P_0 z e^{j\omega t} \end{aligned} \tag{59}$$

where U is a function of r and P_0 is a constant. Then expression (58a) yields

$$\frac{d^2U}{dr^2} + \frac{1}{r}\frac{dU}{dr} - \frac{j\omega\rho}{\eta}U = \frac{P_0}{\eta}. \tag{60}$$

Since the disturbance must not become infinite as

[3] H. Lamb *Hydrodynamics* (Cambridge University Press, Cambridge, 1932), sixth edition, p. 614.
[4] S. Tomotika and T. Aoi, Quart. J. Mech. and Appl. Math. 3, 140–161 (1950).
[5] S. Tomotika and T. Aoi, Quart. J. Mech. and Appl. Math. 4, 401–406 (1951).

$r \to \infty$ the solution of (60) appropriate to the problem is

$$U = AK_0(\beta r) + \frac{jP_0}{\omega\rho}, \tag{61}$$

where

$$\beta = (j\omega/\nu)^{\frac{1}{2}}.$$

Now at $r = r_0$, $U = U_0$ where U_0 is the velocity amplitude of the wire in the direction of its length. Therefore (61) becomes

$$U = \left(U_0 - \frac{jP_0}{\omega\rho}\right)\frac{K_0(\beta r)}{K_0(\beta r_0)} + \frac{jP_0}{\omega\rho}. \tag{62}$$

When $U_0 = 0$ it follows that U must be zero everywhere, under steady state conditions, so that P_0 in expressions (60) must be zero and (62) reduces to

$$U = U_0 K_0(\beta r)/K_0(\beta r_0). \tag{63}$$

The force, F, which the fluid exerts on the wire, per unit length of wire, is then given by

$$F = \mu 2\pi r_0 (\partial v_z/\partial r)_{r_0} \tag{64}$$

or

$$F = -2M'(\omega/\psi) j^{\frac{1}{2}} \frac{K_1(\psi j^{\frac{1}{2}})}{K_0(\psi j^{\frac{1}{2}})} U_0 e^{j\omega t}, \tag{65}$$

where $\psi = (\omega/\nu)^{\frac{1}{2}} r_0$ and M' is the mass of fluid displaced by a unit length of wire. Let

$$\frac{j^{\frac{1}{2}}}{\psi}\frac{K_1(\psi j^{\frac{1}{2}})}{K_0(\psi j^{\frac{1}{2}})} = h' + jh. \tag{66}$$

The added inertia per unit length of wire is then

$$2hM' \tag{67}$$

and the frictional constant, R, per unit length is expressed as

$$R = 2h'\omega M'. \tag{68}$$

The quantities h' and h are tabulated as a function of the variable ψ in Table II. In computing values of K_n the function was expressed in terms of a series of positive powers of the variable. For values of $\psi \geq 4$ an asymptotic expansion for K_n may be used.[6] As $\psi \to 0$, h and h' are given by the following expressions:

$$h = \frac{1}{\psi^2}\frac{\pi/4}{[\log(\psi/2) + \gamma]^2 + (\pi/4)^2}, \tag{69a}$$

$$h' = \frac{1}{\psi^2}\frac{-[\log(\psi/2) + \gamma]}{[\log(\psi/2) + \gamma]^2 + (\pi/4)^2}. \tag{69b}$$

The frictional constant, R, given by (68) and (69) approaches zero as ω or $r_0 \to 0$. The solution for this case is not subject to the limitation imposed on the corresponding solution to the case discussed above since the nonlinear terms in the hydrodynamical equations are identially zero for the case under discussion.

TABLE II.

ψ	ψh	$\psi h'$
0.1	1.1428	3.7485
0.2	0.9350	2.4314
0.3	0.8573	1.9379
0.4	0.8163	1.6732
0.5	0.7916	1.5050
0.6	0.7749	1.3888
0.7	0.7630	1.3030
0.8	0.7542	1.2370
0.9	0.7474	1.1845
1.0	0.7420	1.1416
1.1	0.7377	1.1060
1.2	0.7342	1.0759
1.3	0.7312	1.0501
1.4	0.7288	1.0277
1.5	0.7267	1.0081
1.6	0.7249	0.9908
1.7	0.7233	0.9754
1.8	0.7220	0.9616
1.9	0.7208	0.9491
2.0	0.7197	0.9378
2.5	0.7160	0.8944
3.0	0.7137	0.8647
3.5	0.7122	0.8432
4.0	0.7111	0.8269
∞	zero	zero
0	$\psi^2 h = 0$	$\psi^2 h' = 0$

For values of $\psi \geq 4$, h and h' can be computed from the formulas[6]

$$h = \frac{1}{2^{\frac{1}{2}}\psi}\left[1 + \frac{1}{8\psi^2}\right], \tag{70a}$$

$$h' = \frac{1}{2^{\frac{1}{2}}\psi}\left[1 + \frac{1}{2^{\frac{1}{2}}\psi} - \frac{1}{8\psi^2}\right]. \tag{70b}$$

Now consider a wire immersed in a fluid medium in which a plane acoustic wave is propagating in the direction of the wire as illustrated in Fig. 7(b). If the length of the wire were much smaller than one wavelength and the diameter were much smaller than the length, differential equation (49) could be used to describe the motion. The effective masses and frictional constant can be calculated by employing expressions (67) and (68). In the configuration of interest in this paper, the wire is many wavelengths long. In order to estimate the time rate of conversion of acoustic energy into heat in this case, we will assume that the wire does not move, i.e., $\dot{s} = 0$. In place of expression (50) for Q_v, the amount of acoustic energy converted into heat per second per unit length of wire, we have for this case

$$Q_v = \frac{1}{\tau_0}\int_0^{\tau_0} R\dot{\xi}^2 dt = \frac{U_{00}^2 R}{2} \tag{71}$$

where R is given by expression (68).

[6] G. N. Watson *A Treatise on the Theory of Bessel Functions* (Cambridge University Press, Cambridge 1944), p. 202.

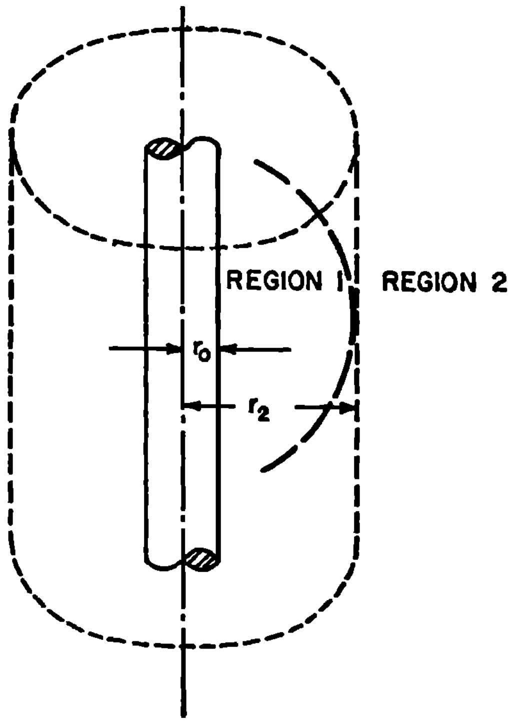

FIG. 8. Thermocouple wire and subdivision of the imbedding medium which are used in the computation of the temperature rise at the wire resulting from the action of viscous forces.

If the angle between the direction of propagation of an acoustic disturbance and the direction of the wire is designated by θ, then the appropriate value Q_v is realized by adding the expressions obtained by inserting for U_{00} in Eqs. (57) and (71) the quantities $U_a \sin\theta$ and $U_a \cos\theta$ respectively. The quantity U_a designates the particle velocity amplitude of the incident wave.

(c) *Temperature rise at the wire.*—A formula for estimating the temperature rise at the wire resulting from the time rate of production of heat Q_v (given by expressions (57) and (71)) will now be obtained.

The disturbance of the fluid resulting from the action of the viscous forces on the wire thermocouple decays rapidly at distances from the wire greater than several diameters, for the range of diameters and acoustic frequencies of interest in this paper. We will, therefore, estimate the magnitude of the temperature rise at the thermocouple by assuming that all of the heat generated by the mechanism under discussion is produced at the boundary of the wire and the fluid. Under conditions such that the disturbance extends with appreciable amplitude to distances of many wire diameters, the temperature change calculated under the assumption that all heat is produced at the boundary between thermocouple and fluid serves as an upper limit. At equilibrium the temperature is maximal and we now obtain an appropriate relation.

The "diameter" of the sound beam determines the length of the wire along which the disturbance resulting from viscous forces extends. Since the beam "diameter" for the cases of interest is large compared to the wire diameter (ratio of 20 or greater) we will choose a cylindrical distribution function for describing the temperature change in the cylindrical region between r_0, the wire radius, and r_2, a radius which is dependent on the size of the beam. At distances from the junction greater than r_2 we choose a spherical distribution function to describe the temperature change. See Fig. 8 for an illustration. The two distribution functions are matched at r_2. This approximate method of analysis is considered sufficiently accurate for obtaining an estimate of the temperature rise at the junction resulting from the action of the viscous forces.

Accordingly we let the temperature distribution in region 1 be described by

$$\Delta T_1 = D_1 \log r + E_1. \tag{72}$$

In region 2 the temperature function is chosen as

$$\Delta T_2 = -(D_2/r) + E_2. \tag{73}$$

If we assume that the wire conducts away no heat then the boundary condition at r_0 is

$$-K(d\Delta T_1/dr)_{r_0} = Q, \tag{74}$$

where Q is the heat generated per unit area per second at the wire boundary by the viscous forces ($Q_v = 2r_0 Q$). As $r\to\infty$ $\Delta T_2\to 0$, and at $r=r_2$ $\Delta T_1=\Delta T_2$. At $r=r_2$ we can also obtain another relation between the constants of expressions (72) and (73) by observing that at equilibrium the amount of heat generated per second at the wire boundary along a length $2r_2$ centered at the junction must equal that passing per second through the spherical surface $r=r_2$. This yields the following approximate relation

$$-K(d\Delta T_2/dr)_{r_2} = \frac{(2r_2)(2\pi r_0)Q}{4\pi r_2^2} = \frac{r_0}{r_2}Q. \tag{75}$$

When the distribution functions (72) and (73) are restricted by the four conditions just noted we obtain the following expression for the temperature rise, ΔT_0, at equilibrium at the thermocouple junction:

$$\Delta T_0 = \frac{Q_v}{2\pi K}[1+\log(r_2/r_0)].\dagger \tag{76}$$

The ratio, γ_0, of this temperature change to that

† If the heat conducted away by the thermocouple wires is taken into account formula (76) is replaced by (this formula applies to the case in which the junction is placed at the peak of the beam used previously as an example)

$$\Delta T_0 = \frac{Q_v}{2\pi K}\left\{1+\log(r_2/r_0)\frac{1-(r_0/L)^2(K_w/K)1.48}{1+(r_0/L)^2(K_w/K)1.48\log(r_2/r_0)}\right\}.$$

In deriving this formula the heat conductivity coefficients of the two wires were chosen equal, (K_w). The quantity L has the units of length and is numerically equal to one when the centimeter is used as the unit of length.

caused by absorption is therefore

$$\gamma_0=\frac{Q_v\rho C}{2\pi K\mu It}[1+\log(r_2/r_0)]. \qquad (77)$$

As a specific example let the wire be oriented transverse to the sound beam. In this case (77) can be written as

$$\gamma_0=\frac{C}{2\pi K\mu tV}R\frac{[1-(1+k)/\{(M/M')+k\}]^2}{[1+(k'/\{(M/M')+k\})^2]}\times[1+\log(r_2/r_0)] \qquad (78)$$

where R is given by expression (52).

Let the diameter of the wire be 0.0005 in. (0.0013 cm) and the temperature of the castor oil imbedding medium be 25 °C ($\eta=6.5$ poises). If the frequency of the acoustic disturbance is 1.0 mc and the duration is 1.0 sec and we choose $r_2/r_0=200$, which is a reasonable value for the beam pattern of the transducer used in obtaining the experimental measurements, we obtain $\gamma_0=0.9$. Heat conduction in the thermocouple wires makes no appreciable change in this calculated value. This value of γ_0 is in agreement with the experimental results given in the accompanying paper.

If the direction of propagation of the sound is along the direction of the wire the above expression (78) is replaced by

$$\gamma_0=\frac{C}{2\pi K\mu tV}R[1+\log(r_2/r_0)], \qquad (79)$$

where R is given by (68). If we again choose $r_2/r_0=200$ we obtain $\gamma_0=2.2$. As indicated previously if the direction of propagation of the sound is neither along the wire nor perpendicular to the wire one can compute the value of γ_0 by first obtaining the components of the particle velocity in these two directions. It is noted that for a sound intensity as high as 100 watts/cm^2 the Reynolds number for this example is only about 0.05.

(*d*) *Time required for the temperature rise to realize a given fraction of the equilibrium value.*—We require an estimate of the time necessary for the temperature rise at the wire, resulting from the time rate of production of heat Q_v (see (57) and (71)), to realize a certain fraction of its equilibrium value. A rough estimate is obtained for the situation in which the junction is located at the beam maximum. Consider a linear array of spherical surface sources of heat each of a diameter equal to the diameter of the thermocouple wires. Adjacent spheres are tangent as illustrated in Fig. 9. The length of the array is determined by the "diameter" of the sound beam. We choose a length of 2 mm for the following calculations. Let the heat produced by each source be the same. The behavior of the temperature change as a function of time at the surface of the middle sphere will be used as an indication of the behavior of the temperature as a function of time at the thermocouple junction. At any arbitrary time after the initiation of heat production, the temperature rise ΔT at the surface of the middle sphere of the linear array can be calculated by using the following formula given by Carslaw and Jaeger[7] for the temperature change resulting from a spherical surface source of heat.

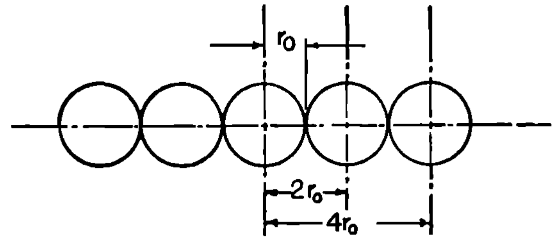

FIG. 9. A linear array of tangential spherical surface sources of heat.

$$\Delta T=\frac{q}{4\pi\mathfrak{k}r}\left\{\frac{(\mathfrak{k}t)^{\frac{1}{2}}}{\pi^{\frac{1}{2}}}\frac{1}{r_0}\times\{\exp[-(r-r_0)^2/4\mathfrak{k}t]-\exp[-(r+r_0)^2/4\mathfrak{k}t]\}-\frac{(r-r_0)}{2r_0}\operatorname{erfc}\left[\frac{r-r_0}{2(\mathfrak{k}t)^{\frac{1}{2}}}\right]+\frac{(r+r_0)}{2r_0}\operatorname{erfc}\left[\frac{r+r_0}{2(\mathfrak{k}t)^{\frac{1}{2}}}\right]\right\} \qquad (80)$$

where $\mathfrak{k}=K/\rho C$, r_0 is equal to the radius of the sphere, q is equal to the time rate of production of heat over the surface of each sphere divided by ρC. At equilibrium, i.e., as $t\rightarrow\infty$, the above expression reduces to

$$\Delta T=\frac{q}{4\pi\mathfrak{k}r}. \qquad (81)$$

For the purpose of obtaining the approximate result desired in this section, we sum the temperature changes resulting from each of the spherical surface sources at distances $r=r_0$, $2r_0$, $4r_0\cdots150r_0$. At the distance r_0 we have the contribution of only one sphere, the middle one, but at all other distances two spheres contribute to the temperature rise. This summation is then taken as an approximate value for the average temperature rise at the surface of the middle sphere. For a wire diameter of 0.0013 cm ($r_0=0.0013/2$) the temperature rise at the surface of the middle sphere, calculated as indicated, realizes 55 percent of its equilibrium value in 0.1 second. The temperature at the surface of the middle sphere of a 4 mm array with equal heating over each sphere realizes 50 percent of its equilibrium value in 0.1 second.

2. *Heat Conduction*

Under the action of an acoustic disturbance heat is transferred across the boundary between the fluid and the wires of the thermocouple. This periodic transfer of heat at the boundary gives rise to a term in the func-

[7] H. S. Carslaw and J. C. Jaeger, *Conduction of Heat in Solids*, (Oxford University Press, London, 1948), p. 222.

tion describing the periodic temperature change in the fluid which is not in phase with the pressure. Hence, acoustic energy is transformed into heat in the fluid which is located in the immediate neighborhood of the boundary. This effect has been discussed by Herzfeld for plane waves incident normally on a plane wall.[8]

In this section an expression for the time rate of conversion of acoustic energy into heat (per unit length of wire) will be obtained for thermocouple wires imbedded in a fluid medium. Since the compressibility of the wires is of the order of 1/20 that of the oil we will assume that the metal wires are incompressible in carrying out the following calculations. Since the diameter of the wires is small compared to one wavelength of the acoustic disturbance, we will assume that the pressure is independent of the space coordinates over the region of the fluid in which the effect considered in this section is important. The following analysis shows that this assumption is justified since the disturbance is shown to be limited to a region of the fluid in the immediate vicinity of the wire. This assumption concerning the pressure considerably simplifies the analysis.

The relation expressing the conservation of energy in the fluid can be written as

$$K\nabla^2(\Delta T)-(C_p/\tilde{V})\partial(\Delta T)/\partial t \\ -[(C_p-C_v)/\tilde{V}][(\partial g/\partial p)/(\partial g/\partial T)]\partial p^0/\partial t=0, \quad (82)$$

where ΔT is the function describing the temperature change in the fluid, $\tilde{V}$ is its molar volume and C_p and C_v are the heat capacities per mole at constant pressure and constant volume respectively. The symbol p^0 designates the time dependent part of the pressure. The relation $g(p, \tilde{V}, T)=0$ is the equation of state for the fluid. The compressibility, ζ, (at constant temperature) and the coefficient of volume expansion, τ, (at constant pressure) of the fluid are expressed in terms of the partial derivatives of the function g as follows:

$$\zeta=(g_p/g_{\tilde{V}})/\tilde{V}, \\ \tau=(-g_T/g_{\tilde{V}})/\tilde{V}. \quad (83)$$

Since the wire is assumed to be incompressible the relation describing the flow of energy is the common heat conduction equation,

$$K_w\nabla^2(\Delta T_w)-C_w'\partial(\Delta T_w)/\partial t=0. \quad (84)$$

Since the space variation of acoustic pressure is small over distances of the order of the wire diameter, the analysis will be restricted to functions describing the temperature changes which are independent of the coordinate along the direction of the wire.

Let ΔT of (82) and ΔT_w of (84) designate only the periodic temperature change, i.e.,

$$\Delta T=\Delta T' e^{j\omega t}, \\ \Delta T_w=\Delta T_w' e^{j\omega t}, \quad (85)$$

[8] K. F. Herzfeld, Phys. Rev. 53, 899–906 (1938).

and let

$$p^0=p' e^{j\omega t}. \quad (86)$$

From relations (82) and (84) we obtain

$$K\nabla^2(\Delta T')-j\omega(C_p/\tilde{V})(\Delta T') \\ +j\omega[(C_p-C_v)/\tilde{V}][-g_p/g_T]p'=0 \quad (87)$$

and

$$K_w\nabla^2(\Delta T_w')-j\omega C_w'(\Delta T_w')=0. \quad (88)$$

Since p' is independent of the space coordinate, we obtain the following solution to (87); $\Delta T'$ must approach zero as $r\rightarrow\infty$,

$$\Delta T'=Ap'+GK_0(mr), \quad (89)$$

where

$$m^2=(j\omega/K)(C_p/\tilde{V}) \quad (90)$$

and

$$A=\frac{C_p-C_v}{C_p}(-g_p/g_T). \quad (91)$$

The solution to (88) is

$$\Delta T_w'=HI_0(nr), \quad (92)$$

where

$$n^2=j\omega C'_w/K_w. \quad (93)$$

The boundary conditions at $r=r_0$ are

$$\Delta T'=\Delta T_w', \\ K(d(\Delta T')/dr)_{r_0}=K_w(d(\Delta T_w')/dr)_{r_0}. \quad (94)$$

These conditions impose the following relations on H and G:

$$Ap'+GK_0(mr_0)=HI_0(nr_0), \quad (95)$$

$$KmGK_0'(mr_0)=K_w nHI_0'(nr_0). \quad (96)$$

The solution for G is

$$G=-Ap'/BK_0(mr_0), \quad (97)$$

where

$$B=(K/K_w)(m/n)(K_1(mr_0)/ \\ K_0(mr_0))(I_0(nr_0)/I_1(nr_0))+1 \quad (98)$$

where use has been made of the relations $K_0'=-K_1$, and $I_0'=I_1$. The periodic temperature change in the fluid as described by (89) is now completely determined. As the next step in the determination of the rate of conversion of acoustic energy into heat, we require an expression for the periodic volume change in the liquid. This can be obtained by using the equation of state in conjunction with (89). From the equation of state we obtain

$$\Delta\tilde{V}/\tilde{V}=\tau\Delta T-\zeta p^0. \quad (99)$$

Upon substituting into (99) the expression for $\Delta T'$ given by (89) we obtain the volume change in terms of the pressure

$$\frac{\Delta\tilde{V}}{\tilde{V}}=p'\left\{\tau A\left[1-\frac{K_0(mr)}{BK_0(mr_0)}\right]-\zeta\right\}e^{j\omega t}. \quad (100)$$

The work done on an element of the fluid of volume v during one period of the acoustic disturbance is given by the integral

$$-\int_0^{t_0} pR\left[\frac{dv}{dt}\right]dt, \quad (101)$$

where t_0 is equal to one period of the acoustic disturbance, $p=p_0+p'\cos\omega t$ and $R[dv/dt]$ represents the real part of dv/dt. An expression for dv/dt can be obtained from (100). We note that $\Delta v/v_0=\Delta\tilde{V}/\tilde{V}$ so that (100) yields

$$v=v_0+\Delta v=v_0+v_0p'\left\{\tau A\left[1-\frac{K_0(mr)}{BK_0(mr_0)}\right]-\zeta\right\}e^{j\omega t}. \quad (102)$$

Thus

$$\frac{dv}{dt}=j\omega v_0p'\left\{\tau A\left[1-\frac{K_0(mr)}{BK_0(mr_0)}\right]-\zeta\right\}e^{j\omega t}. \quad (103)$$

We note that in the absence of heat conduction

$$R[dv/dt]=0$$

and the integral (101) is therefore zero, i.e., no acoustic energy is converted into heat.

If we perform the integration indicated by (101) the following result is obtained

$$-\pi v_0p'^2Rj\left\{\tau A\left[1-\frac{K_0(mr)}{BK_0(mr_0)}\right]-\zeta\right\}. \quad (104)$$

The symbol R designates the real part of the complex expression. Let $v_0=2\pi rh_ldr$ and integrate over r from from r_0 to ∞ in order to obtain the work done throughout the fluid. Since the disturbance resulting from heat conduction dies out very rapidly as one moves away from the wire (the second term of expression (89)), practically the entire contribution to the integral is furnished over a range of values of r close to r_0, and one need not be concerned with the fact that the distance from r_0 to ∞ is not less than one wavelength. The quantity h_l is the length in the direction of the wire of the column of fluid under consideration. This integral can be written as

$$2\pi^2p'^2\tau Ah_lR\left\{\frac{j}{BK_0(mr_0)}\int_{r_0}^{\infty} rK_0(mr)dr\right\}. \quad (105)$$

If the fluid is castor oil and the acoustic frequency is 1.0 mc then $|m^2|=0.7\ (10)^{10}\ \text{cm}^{-2}$. If $r_0\geq 5\ (10)^{-4}$ cm then $|mr_0|\geq 30$ and $K_0(mr)$ can be approximated closely by the expression $(\pi/2mr)^{\frac{1}{2}}e^{-mr}$. We then obtain

$$\int_{r_0}^{\infty} rK_0(mr)dr\simeq(\pi/2)^{\frac{1}{2}}[(mr_0)^{\frac{1}{2}}e^{-mr_0} + (\pi^{\frac{1}{2}}/2)\ \text{erfc}(mr_0)^{\frac{1}{2}}]/m^2. \quad (106)$$

When $|(mr_0)^{\frac{1}{2}}|\geq(40)^{\frac{1}{2}}$ the erfc function is given approximately as follows:

$$(\pi^{\frac{1}{2}}/2)\ \text{erfc}(mr_0)^{\frac{1}{2}}\simeq e^{-mr_0}/2(mr_0)^{\frac{1}{2}}. \quad (107)$$

The symbol W designates the time rate at which work is done on the fluid per unit length in the direction of the wire and is equal to the product of (105) and the frequency of the sound divided by h. The following result is obtained, $K_0(mr_0)$ has been approximated as indicated above.

$$W=\pi\tau(p')^2[C_p-C_v/C_p]\times[K/(C_p/\tilde{V})]R\{(mr_0+\tfrac{1}{2})/B\}. \quad (108)$$

It is of interest to compare this result with that for the time rate of conversion of acoustic energy into heat resulting from the action of the viscous forces given by expressions (57) and (71). We form the ratio W/Q_v where Q_v is given by (71). Use of expression (57) in place of (71) yields a result numerically of the same order for the wire sizes of interest in this paper. An expression for the ratio W/Q_v follows:

$$\frac{W}{Q_v}=\frac{\pi\tau(\rho V)^2}{h'\omega M'}\left(\frac{C_p-C_v}{C_p}\right)\left(\frac{K}{C_p/\tilde{V}}\right)R\{(mr_0+\tfrac{1}{2})/B\}. \quad (109)$$

For a wire of copper or constantan of a diameter 0.0013 cm and for castor oil as the imbedding medium the real part of B is approximately equal to one. The imaginary part of B is small compared to one. It follows that since the argument of mr_0 is $\pi/4$ we can obtain an approximate expression for W/Q_v by letting $B=1$ in (109). Relation (109) yields for a wire of this diameter in castor oil about $1.4\ (10)^{-3}$ for the ratio W/Q_v.

III. SUMMARY

The formula basic to the determination of absolute sound levels or acoustic absorption coefficients by the method analyzed in this paper is

$$\mu I=\rho C\left(\frac{dT}{dt}\right)_0. \quad (110)$$

The evaluation of the quantity $(dT/dt)_0$ from the experimentally measured temperature-time relation yielded by the thermocouple can be readily accomplished if certain criteria are satisfied.

The finite, nonzero, heat capacity of the thermocouple wires and the effect of heat conduction away from the junction by the wires impose limitations on the wire diameter. Symbolically

$$\frac{\delta T_e}{\Delta T_m}=\frac{3r_0^2}{2t}\left(\frac{\rho_wC_w}{K}\right)(100) \quad (111)$$

and

$$\frac{\delta T}{\Delta T_m}=-\frac{3r_0^2}{2}\left(\frac{K_w}{K}\right)\frac{f''(x)}{f(x)}(100) \quad (112)$$

yield the respective fractional uncertainties in $(dT/dt)_0$. For 0.0005 in. diameter copper imbedded in castor oil and for a one second pulse of sound, beam width at half intensity equal to or greater than 4 mm, the calculated uncertainties are of the order of 0.1 percent and 0.5 percent.

The dependence of the magnitude of the quantity $\mu/\rho C$ on the temperature causes a deviation of the temperature-time functional relation from the linear form (absorption in the interior of the liquid). If

$$\left[-\frac{\partial}{\partial T}(\mu/\rho c)/(\mu/\rho c)\right]_{T0}\Delta T_c$$

is of the order of 0.1 or less the fractional deviation from linearity can be expressed as

$$\frac{\delta T_c}{\Delta T_c}=\frac{\Delta T_c}{2}\left[\frac{\partial/\partial T(\mu/\rho C)}{\mu/\rho C}\right]_{T_0}, \tag{113}$$

where T_0 designates the temperature of the absorbing medium before irradiation. At a frequency of 1.0 mc for castor oil at 25°C and a temperature rise of 1°C the ratio is equal numerically to about 0.04. Relation (113) is used in the processing of experimental data to accurately evaluate the quantity $(dT/dt)_0$ of expression (110).

The process of heat conduction in the absorbing medium limits the accuracy of intensity determinations (or absorption coefficients) as the beam width becomes small. An expression for the fractional uncertainty is

$$\frac{\delta T_d}{\Delta T_d}=-\frac{K}{\rho C}\left[\frac{f''(r)}{f(r)}\right]_{r=0}t. \tag{114}$$

For castor oil and a beam width of 4 mm the calculated uncertainty is of the order of 0.3 percent for a 1.0 second period of irradiation.

The uncertainty introduced by variation in temperature in the wire as a function of the radial coordinate is calculated and shown to be negligible for pulse lengths of the order of one second.

In addition to the temperature change which occurs as a result of the conversion of acoustic energy into heat in the body of the absorbing medium the temperature also changes in the neighborhood of the thermocouple because of viscous force action between the wires and the imbedding medium. The magnitude of the temperature rise resulting from this action is of the same order as the temperature change resulting from absorption for wire sizes and pulse durations of practical interest. However, this temperature change can be separated from that caused by absorption in the interior of the fluid because of its rapid approach to equilibrium. The magnitude of the temperature rise relative to that caused by absorption can be estimated from expressions (78) and (79) of the analysis. For 0.0005 in. diameter wire in castor oil, at 25°C, frequency 1.0 mc and a pulse duration of 1.0 second, the ratio is of the order of unity.

The transfer of heat across the boundary between the wire and the imbedding medium during an acoustic disturbance results in a conversion of acoustic energy into heat and a concomitant temperature change. For the specific example used as an illustration in the previous paragraph, calculation shows (expression (109)) that this effect produces a temperature change of the order of 1/1000 that produced by viscous force action.

ACKNOWLEDGMENT

The authors thank Mr. Wayne Peterson for performing the numerical calculations for Table II.