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Mechanism of Acoustic Absorption in Tissue*

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A viscous mechanism for the absorption of ultrasonic energy in tissue is considered. It is shown that the viscous forces acting between a suitably chosen distribution of suspended particles or structure elements and a suspending liquid can account for the experimentally observed linear relations between acoustic absorption coefficient and ultrasonic frequency. The frequency band over which linearity obtains is determined by the limits of the distribution of values for the parameters which describe the structure elements. Below the linear range the relation becomes quadratic, in agreement with experiment, and at "high" frequencies a limiting value is approached asymptotically.

1. INTRODUCTION

IT is the purpose of this paper to suggest how a linear relationship between the acoustic absorption coefficient per unit path length and the frequency might be explained for biological tissue. The measurements of T. Hüter in the frequency range from 1.5 mc to 4.5 mc for various tissues show that the absorption coefficient is directly proportional to the frequency in this range.[1]

The frequency dependence of the absorption coefficient has been evaluated for a number of different mechanisms, and the results constitute an extensive literature. In a recent review article Kittel discusses these various mechanisms.[2] Several which might appear to require consideration from the viewpoint of understanding the process of absorption of ultrasonic energy in biological tissue are scattering, relaxation, and viscous absorption. The term "relaxation" refers to processes in which the time intervals required for energy exchange between various degrees of freedom of a system are not negligible compared with the period of the acoustic disturbance.

If Rayleigh scattering were the principal cause of ultrasonic absorption in tissue one would expect that the absorption coefficient per unit path length would be proportional to the fourth power of the frequency.[3,4] Relaxation effects, for materials with single relaxation frequencies, yield a quadratic dependence at low frequencies, a constant value at high frequencies, and no intermediate band which corresponds to the experimental data on tissue. Calculations based on the action of the viscous forces to convert acoustic energy into heat show that in a homogeneous medium the absorption coefficient is proportional to the square of the frequency.[5] It has also been shown that the viscous forces acting between a fluid medium and a suspension of uniform particles in the medium yields a relaxation type of frequency dependence of the absorption coefficient.[6,7] It is also of interest to note that a theory for the viscoelastic characteristics of rubber has been advanced which treats the rubber chains as springs in a viscous medium.[8]

In the following analysis it will be shown that the viscous forces acting between a suitably chosen distribution of suspended particles or structure elements and a suspending liquid can account for a linear relation between the acoustic absorption coefficient (per unit path length) and the frequency over a wide frequency range.

2. ANALYSIS

Since we are primarily interested in indicating the type of mechanism which might account for the observed dependence of the acoustic absorption coefficient (per unit path length) of tissue on frequency,[1] we will consider in this paper only a relatively simple case. Extensions of the theory, if further experimental work substantiates the general picture, would then be in order.

In the following discussion the term "structure element" is used to designate a constituent of the tissue, either intercellular or intracellular, whose motion in the sound field can be described by a single value of displacement. In the analysis coupling between structure elements is not considered. However, coupling is readily incorporated into the theory if further experimental work appears to require such an extension.

Consider a structure element of mass M suspended in a liquid of density ρ_0. Assume that the frictional force exerted on the element by the liquid is $R(\dot{\xi}-\dot{r})$, where $\dot{\xi}$ is the velocity of the liquid, $\dot{r}$ is the velocity of the element, and R is a constant (for any one element) which depends upon the structure and orientation of the element. The analysis is restricted to consideration of a one-dimensional problem and the following differential equation given by Hiedemann is used to describe the motion of the element. (See, for example, the paper of

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[1] T. Hüter, Naturwiss. 35, 285 (1948).

[2] C. Kittel, *Reports on Progress in Physics* 9, 205 (1948).

[3] W. P. Mason and H. J. McSkimin, J. Acoust. Soc. Am. 19, 464 (1947).

[4] Lord Rayleigh, *Theory of Sound* (Dover Publications, New York, 1945), second edition.

[5] See, for example, reference 4.

[6] See Bergmann's book, *Der Ultraschall*, third edition, published by J. W. Edwards (1944) for a review of the literature to 1942. Brandt, Freund and Hiedemann have published a number of papers on this subject.

[7] Angerer, Barth and Güttner, Strahlentherapie 84, 601 (1951).

[8] R. B. Blizard, J. Appl. Phys. 22, 730 (1951).

Angerer, Barth, and Güttner, reference 7.)

$$\ddot{r}+\frac{R}{M_e}\dot{r}=\frac{R}{M_e}\dot{\xi}+\frac{m_e}{M_e}\ddot{\xi}. \qquad (1)$$

The quantity M_e is the "effective" mass of the element and m_e is the "effective" mass of an element of the suspending liquid identical in shape and size to the structure element. We note that for a rigid sphere of radius a, the "effective" mass of the element is equal to the sum of the mass M of the sphere and the mass m of the displaced liquid multiplied by the factor f, where[9]

$$f=(1/2)+(9/4a)(2\eta/\omega\rho)^{\frac{1}{2}}.$$

The quantity η is the coefficient of viscosity of the liquid and ρ is its density. In symbolic form

$$M_e=M+fm,$$
$$m_e=m+fm.$$

We do not suggest that the above expression for f is applicable to structure elements in tissues. In fact, in the discussion following the analysis, we assume that M_e is independent of frequency over the frequency range of interest in this paper.

Equation (1) is consistent with the analysis of acoustic propagation in a homogeneous medium for when $M_e=m_e$ and under the initial conditions $\dot{r}=\dot{\xi}$ and $r=\xi$ it follows from (1) that $r=\xi$ for greater values of the time. Let the velocity $\dot{\xi}$ result from the propagation of an acoustic disturbance through the medium. We suppose that the displacement, r, of the structure element from its equilibrium position is small enough to enable us to neglect the variation of $\dot{\xi}$ with different space positions of the element. That is, we let

$$\dot{\xi}=U\sin\omega t. \qquad (2)$$

The steady state solution of (1) is

$$\dot{r}=U\left[\frac{(\omega^2 m_e/M_e)+(R/M_e)^2}{\omega^2+(R/M_e)^2}\sin\omega t+\frac{(\omega R/M_e)[(m_e/M_e)-1]}{\omega^2+(R/M_e)^2}\cos\omega t\right]. \qquad (3)$$

The amount of acoustic energy converted into heat per cycle per structure element is

$$\int_0^\tau R(\dot{r}-\dot{\xi})^2dt=U^2R\frac{\tau}{2}\frac{\omega^2[(m_e/M_e)-1]^2}{\omega^2+(R/M_e)^2}. \qquad (4)$$

It follows immediately from (4) that the acoustic energy, E, converted into heat per second per unit volume is expressible as follows

$$E=\frac{U^2nR\omega^2[(m_e/M_e)-1]^2}{2[\omega^2+(R/M_e)^2]}, \qquad (5)$$

[9] H. Lamb, *Hydrodynamics* (Cambridge University Press, London, England, 1932), sixth edition, Chap. XI.

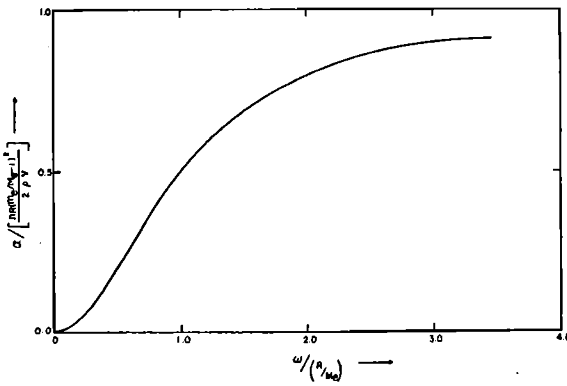

FIG. 1. Relation between absorption coefficient per unit path length and frequency for a suspension of structure elements of identical characteristics.

where n represents the number of particles or structure elements per unit volume having "effective" mass, M_e, displacing an amount of liquid of effective mass, m_e, and having a frictional constant, R.

In terms of the pressure absorption coefficient (per unit path length), α, this quantity E is given as

$$E=\alpha U^2\rho V, \qquad (6)$$

where ρ is the density of the tissue and V is the velocity of sound in the tissue. If one combines (5) and (6), one obtains

$$\alpha=\frac{nR\omega^2[(m_e/M_e)-1]^2}{2\rho V[\omega^2+(R/M_e)^2]}. \qquad (7)$$

A relation of this type has been obtained by the investigators of reference 6. This relation is illustrated in Fig. 1. For low values of ω the absorption coefficient is proportional to the square of the frequency. As ω becomes large the absorption coefficient approaches the limiting value

$$\frac{nR[(m_e/M_e)-1]^2}{2\rho V}. \qquad (8)$$

From the graph it can be seen that there is no frequency range with a band width as large as three to one over which a relation of the form $\alpha=k\omega$ holds reasonably well.

The above expression (7) is applicable to the case in which all elements are characterized by equal values of M_e, m_e, and R. We now consider the case in which structure elements with various values of M_e, m_e, and R are imbedded in the medium. In general, we must consider a distribution over all three parameters.

The form of the distribution function affects the relationship between α and ω. It is convenient to consider a distribution over the parameters M_e, m_e, and R/M_e instead of M_e, m_e, and R. We now show that it is possible to choose a distribution function, not unreasonable physically, which used to generalize (7)

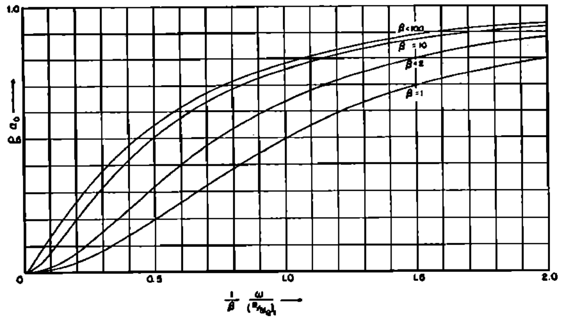

FIG. 2. Relationship between α_0, which is proportional to the absorption coefficient per unit path length (see expression (18a) of text), and the quantity $\omega/\beta(R/M_e)_1$ which is proportional to the frequency ($\omega=2\pi f$). The quantity β is determined by the range of values of the frictional constants and effective masses of the structure elements in the tissue.

yields the desired linear relation between the acoustic absorption coefficient and the frequency.

To calculate the acoustic energy, E, converted into heat per second per unit volume for a distribution of structure elements one must specify the functional dependence of n on the parameters R/M_e, m_e, and M_e. We choose

$$n=\frac{g(M_e, m_e)}{R/M_e}. \tag{9}$$

We obtain for the energy converted per second per unit volume

$$E=\frac{U^2\omega^2}{2}\int\int\left\{g(M_e, m_e)M_e[(m_e/M_e)-1]^2 \times\int\frac{d(R/M_e)}{\omega^2+(R/M_e)^2}\right\}dM_e dm_e, \tag{10}$$

where the limits are equal to the maximum and minimum values of the parameters M_e, m_e, and R/M_e. The form of the function $g(M_e, m_e)$ will not concern us here since it need not be specified in the following argument.

Since there may be a wide range of values of R/M_e for the particles or structure elements in some tissues, we first evaluate the integral over R/M_e with the limits zero and infinity. (This requires an infinite number of particles per unit volume.) We obtain

$$\int_0^\infty\frac{d(R/M_e)}{\omega^2+(R/M_e)^2}=\frac{\pi}{2\omega}.$$

When (10) is combined with (6) the following expression for α results:

$$\alpha=\frac{\pi\omega}{4\rho V}\int\int g(M_e, m_e)M_e[(m_e/M_e)-1]^2 dM_e dm_e. \tag{11}$$

The integral is independent of ω so that the acoustic absorption coefficient (per unit path length) α is directly proportional to the acoustic frequency.

It is desirable to investigate modifications of this result if the range of integration over (R/M_e) is finite and does not include zero. In place of the value $\pi/2\omega$ obtained previously for the integral over R/M_e we now obtain

$$\frac{1}{\omega}\left\{\tan^{-1}\frac{(R/M_e)_2}{\omega}-\tan^{-1}\frac{(R/M_e)_1}{\omega}\right\}, \tag{12}$$

where $(R/M_e)_2$ and $(R/M_e)_1$ are, respectively, the maximum and minimum values of the quantity, R/M_e, exhibited by the structure elements in the tissue.

If

$$(R/M_e)_1/\omega\leqq 0.1 \quad\text{and}\quad (R/M_e)_2/\omega\geqq 10, \tag{13}$$

we express (12) in the approximate form

$$\frac{1}{\omega}\left\{\frac{\pi}{2}-\frac{1}{(R/M_e)_2/\omega}-\frac{(R/M_e)_1}{\omega}\right\}. \tag{14}$$

This is approximately equal to $\pi/2\omega$, and as before we obtain α proportional to ω.

Two limiting cases of (12) are of interest. As $\omega\rightarrow\infty$ (12) approaches

$$\frac{1}{\omega^2}\left\{\left(\frac{R}{M_e}\right)_2-\left(\frac{R}{M_e}\right)_1\right\}, \tag{15}$$

and α is independent of frequency. As $\omega\rightarrow 0$, (12) becomes

$$\left\{\frac{1}{(R/M_e)_1}-\frac{1}{(R/M_e)_2}\right\}, \tag{16}$$

and α is proportional to the square of the frequency.

T. Hüter has just recently completed a series of measurements of the ultrasonic absorption coefficients of bone and nerve tissue as a function of frequency. These new results, which are in the process of publication,[10] are in quantitative agreement with the theory developed in this paper.

The quadratic dependence of α on frequency as $\omega\rightarrow 0$ is in agreement with the work of another group of investigators.[11]

It is expected that mechanisms other than the one discussed in this paper would be of primary importance in determining acoustic absorption in some regions of the spectrum.

3. DISCUSSION

It is shown by the above analysis that a suitably chosen distribution function of the parameter R/M_e (viscous force constant divided by the mass) for the

[10] T. Hüter, Naturwiss. (to be published). See also M.I.T. Quart. Prog. Rept. (July-September, 1951).

[11] Franke, von Gierke, Oestreicher, Parrack, von Wittern. For a summary publication of some of their work see A. F. Technical Report No. 6367 (February 1951).

particles or structure elements present in biological tissue can account for a linear relation between the acoustic absorption coefficient (per unit path length) and the frequency. The frequency interval over which such a relation holds is determined by the maximum and minimum values of the ratio R/M_e for the elements present in the medium. The graph of Fig. 2 indicates this dependence of the frequency interval of linearity on the ratio of the maximum to minimum values of the quantity R/M_e. The quantities β and α_0 of the graph are defined by the equations

$$\beta=\frac{(R/M_e)_2}{(R/M_e)_1}, \tag{17}$$

$$\alpha_0=\frac{\alpha}{(R/M_e)_1(\beta-1)(\frac{1}{2}\rho V)} \times\int\int g(M_e,m_e)M_e[(m_e/M_e)-1]^2dM_edm_e. \tag{18a}$$

From (18a) and results derived above we obtain

$$\alpha_0=\frac{\omega}{(\beta-1)(R/M_e)_1} \times\left\{\tan^{-1}\beta\frac{(R/M_e)_1}{\omega}-\tan^{-1}\frac{(R/M_e)_1}{\omega}\right\}. \tag{18b}$$

The curve for $\beta=1$ corresponds to the case of a suspension of structure elements all having the same value of the ratio R/M_e. This curve is identical with the curve of Fig. 1. The linear portion extends over an interval of about three to one. The intercept of the linear portion on the frequency axis is numerically about $\frac{1}{4}$ as large as the midfrequency of the linear portion. The curve $\beta=10$ contains a linear portion of about six to one with an intercept numerically about $\frac{1}{6}$ as large as the midfrequency of the linear portion. The curve for $\beta=100$ contains a linear portion with an interval of about twenty to one, and the intercept value on the frequency axis is only a few percent of the midfrequency value of the linear portion. In general, if the linear relation is to hold over a frequency range of the order of $(10)^m$, $m\geqq 1$, then the ratio of the maximum value to the minimum value of R/M_e is of the order of $(10)^{m+1}$.

As a specific example, if the relation between absorption coefficient and frequency is linear to 10 mc, we find that the maximum value of R/M_e must be at least of the order of $0.6\ (10)^8\ \text{sec}^{-1}$. It is of interest to compute the value of R/M_e for a rigid spherical particle on the basis of certain formulas first obtained by Stokes (see reference 9, pp. 643–644). Consider a particle of radius 0.1 micron, $a=(10)^{-5}$ cm, of density 1 gram/cm^3, and water as the fluid medium ($\eta\simeq 0.01$ poise). We use the following formulas for computing R and M_e (letting $\omega=2\pi(10)^7$):

$$R=6\pi a\eta[1+(\omega\rho/2\eta)^{\frac{1}{2}}a]$$

and

$$M_e=M+m[(1/2)+(9/4a)(2\eta/\omega\rho)^{\frac{1}{2}}].$$

We obtain the approximate value $1.3\ (10)^8\ \text{sec}^{-1}$ for the ratio R/M_e. The fact that these numerical values do not differ by several orders of magnitude lends some support to the view that the mechanism discussed in this paper is of importance in the process of absorption of ultrasound in tissue. However, it is not implied that the above formulas for R and M_e are suitable for calculating quantitative values for these parameters for spherical structure elements in tissue.

Structure oriented in specific directions in the tissue could account for a variation in absorption coefficient with direction of propagation since the value of the ratio R/M_e depends upon the direction of propagation of the sound relative to the structures.

We note that, in general, relaxation mechanisms yield a linear dependence of acoustic absorption coefficient on frequency if one assumes formally a distribution over relaxation frequencies of the form (9) above.

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