CHAPTER 2

ACOUSTIC EQUATIONS

Acoustic pressure, intensity, and force are three parameters that are investigated in this study. A discussion of pressure will help to describe intensity, which will in turn help to describe radiation force.

2.1. Pressure

The compressional and rarefactional pressures can be best described in conjunction with each other. In a sinusoidal wave, the compressional pressure, p_c is the maximum part of the wave and the rarefactional pressure, p_r is the minimum part of the wave. These can also be described as p_c being the positive pressure and p_r being the negative pressure. The positive part of a sinusoidal wave can be described as the compressional pressure and the negative part can be described as the rarefactional pressure as shown in Figure 2.1.

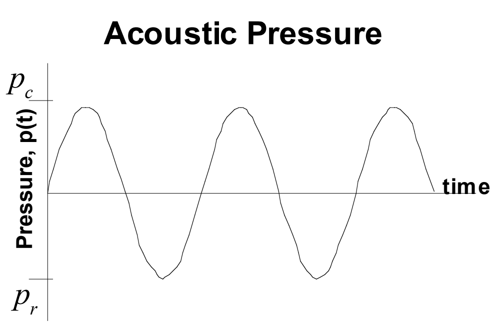

Figure 2.1 Sinusoidal Wave with Acoustic Pressures, p_c and p_r Denoted

2.2 Intensity

Intensity is mainly used when discussing traveling plane waves in a liquid. The definition of intensity is the rate per unit area in which work is being done. When describing sound waves, the instantaneous acoustic intensity is the instantaneous rate per unit area where work is being done and can be defined mathematically as

$$I(t) = p(t)u(t) \tag{2.1}$$

where p(t) is the instantaneous acoustic pressure and u(t) is the instantaneous particle speed. For a plane wave, the equation for I(t) can also be written

$$I(t) = \frac{p^2(t)}{\rho c}. \tag{2.2}$$

The density of the medium is defined as ρ and has the units of $\frac{kg}{m^3}$. The speed of propagation in the medium is defined as c and has units of $\frac{m}{s}$. The product of the density and speed result in the impedance Z, which has units of $\frac{Pa \cdot s}{m}$.

When determining the time averaged intensity, I(TA) the mathematical expression for the pulse averaged intensity I(PA) is used. The time averaged intensity is the average intensity calculated over the time between pulses. The pulse averaged intensity is the average intensity during a pulse. The mathematical expression for I(PA) is

$$I(PA) = \frac{PII}{PD} \tag{2.3}$$

where PII is the pulse intensity integral (described in Section 2.3) and PD is the pulse duration, the time in which the pulse is on. From this I(TA) can mathematically written as

$$I(TA) = I(PA) * \tau \quad (2.4)$$

τ is the duty factor. The duty factor is the ratio of the time when the pulse is on to the total time. For a periodic wave, τ is the ratio of the PD to the pulse repetition period (PRP) and is mathematically defined in equation 2.5.

$$\tau = \frac{PD}{PRP} \quad (2.5)$$

Pulse repetition frequency (PRF) is the inverse of the PRP. Therefore, equation 2.5 can be rewritten as

$$\tau = PD * PRF \quad (2.6)$$

Combining equations 2.3, 2.4, and 2.6, the specific equation for I(TA) can be written as

$$I(TA) = \frac{PII}{PD} * PD * PRF \quad (2.7)$$

More simply, it can be written as

$$I(TA) = PII * PRF \quad (2.8)$$

2.3 Pulse Integral Intensity (PII)

PII is defined as the time integral of the intensity of a pulse taken over the time where the acoustic pressure is nonzero [*Raum and O'Brien*, 1997]. Its mathematical equation is

$$PII = \int_{t_1}^{t_2} I(t)dt \quad (2.9)$$

where t_1 and t_2 the time of the pulse. The value for t_1 is the time just before when the pulse begins. The value of t_2 is just after when the pulse ends. These time values vary depending on the properties of the pulse. A figure of a pulse, $p(t)$ was shown in Figure 2.1. A figure of this pulse squared, or $p^2(t)$ is shown in Figure 2.2.

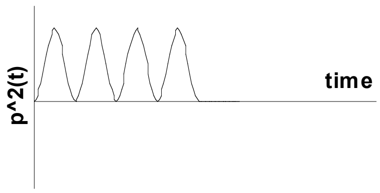

Figure 2.2 Graph of $p^2(t)$

Combining equations 2.2 and 2.9 yield

$$PII = \int_{t_1}^{t_2} I(t)dt = \int_{t_1}^{t_2} \frac{p^2(t)}{\rho c} dt \tag{2.10}$$

Because ρc is independent of time, PII can be simplified to

$$PII = \frac{1}{\rho c} \int_{t_1}^{t_2} (p(t))^2 dt \,. \tag{2.11}$$

Using the equation above, a graph of the PII can be shown in Figure 2.3.

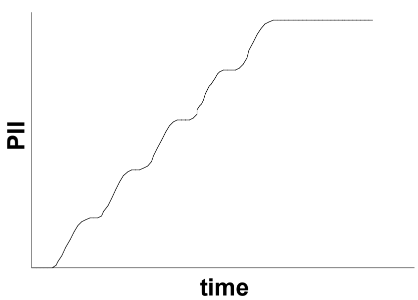

Figure 2.3 Graph of PII

Acoustic pressure is first measured as a voltage. Then, it is translated into a pressure using a pre-established pressure-voltage relation (calibration factor). This calibration factor is specific for each individual device (hydrophone) that is measuring the voltage. K is the symbol used to represent the calibration factor and has units of Volts/Pascal. The equation that relates the pressure and voltage by the calibration factor is

$$p(t) = \frac{v(t)}{K} \tag{2.12}$$

Using equation 2.12, equation 2.11 can be rewritten as

$$PII = \frac{1}{\rho c}\int_{t_1}^{t_2} (\frac{v(t)}{K})^2 dt \tag{2.13}$$

where v(t) has units of volts. Once again, taking the constant out of the integral, the equation simplifies to

$$PII = \frac{1}{\rho c K^2}\int_{t_1}^{t_2} (v(t))^2 dt \tag{2.14}$$

The $\int_{t_1}^{t_2} (v(t))^2 dt$ in the above equation simplifies to a voltage intensity integral, or VII. Thus, the final equation for PII can be written as

$$PII = \frac{VII}{\rho c K^2} \tag{2.15}$$

2.4 Radiation Force

Lord Rayleigh laid the foundation for the theoretical description of radiation force in 1978, after it had first been observed by Dvo_ak in 1876 [*Zieniuk and Chivers*, 1976]. Rayleigh derived the result of radiation force for plane, isothermal waves and it is given by

$$\Pi = \frac{g}{c} ES \tag{2.16}$$

where g is the group velocity, c is the phase of the velocity, E is the energy density and S is the area perpendicular to the wave direction [*Lord Rayleigh*, 1902].

Radiation force is a vector quantity. It can be misnamed as radiation pressure because of the theoretical assumption of planar wave fields [*Zieniuk and Chivers*, 1976]. The mechanism of radiation force on an object can be defined as the integral of the acoustic energy density, E, over the surface of the object [*Beissner,* 1986]. Energy density is defined mathematically as

$$E = \langle E_i \rangle_T = \frac{1}{T}\int_0^T E_i dt \tag{2.17}$$

[*Kinsler,* 2000] where T is defined as the period of a harmonic wave. E_i is the instantaneous energy density. For a planar wave, energy density can also be defined as

$$E = \frac{I(TA)}{c} \tag{2.18}$$

This equation will be more useful and applicable to the experiments performed in this thesis. The time averaged intensity from equation 2.18 can be defined as the product of PII and PRF (equation 2.8), such that equation 2.18 becomes

$$E = \frac{PII * PRF}{c} \tag{2.19}$$

Using the equation above for PII, (2.11), equation 2.19 can be rewritten as

$$\frac{PII * PRF}{c} = \frac{PRF}{\rho c^2}\int_{t_1}^{t_2} (p(t))^2 dt \tag{2.20}$$

This equation can be rewritten as

$$\frac{PII * PRF}{c} = \frac{PRF}{\rho c^2}\int_{t_1}^{t_2} (\frac{v(t)}{K})^2 dt \tag{2.21}$$

by using the relation of K to rewrite $(p(t))^2$ as $(\frac{v(t)}{K})^2$. This equation can be simplified to

$$\frac{PII * PRF}{c} = \frac{PRF}{\rho c^2 K^2}\int_{t_1}^{t_2} (v(t))^2 dt \tag{2.22}$$

From equation 2.22, the integral $\int_{t_1}^{t_2} (v(t))^2 dt$ can be recognized as the VII and can be written as

$$\frac{PII * PRF}{c} = \frac{VII * PRF}{\rho c^2 K^2} \qquad (2.23)$$

This equation can be checked by dividing equation 2.15 by c and multiplying it by PRF to get the same result as in 2.23. Thus, the radiation force can be rewritten as

$$E = \frac{VII * PRF}{\rho c^2 K^2} \qquad (2.23)$$