APPENDIX A

DERIVATION OF PRESSURE EXPRESSIONS

We derive the expression for the far-field pressure produced by an array of finite sources with weighting for sidelobe reduction and with electronic phase shifts for beamsteering. Some simplifications are made to arrive at the equations used in Chapter 2. Also, we show that apodization for lowering sidelobes can be used with amplitude-weighting for beamsteering.

Geometry and variables are defined in Figure A.1. There are N elements of the array, labelled $\frac{-(N-1)}{2}$ to $\frac{N-1}{2}$. Each element is a line source of length a. The interelement spacing is d, measured as the distance between the center points of two adjacent elements. The signals from each element are weighted with a factor $w(n)$ to lower sidelobes, and phase-shifted with a factor $e^{j2n\phi}$ to steer the main lobe to θ_0, where $\phi = \frac{k_0 d}{2} sin(\theta_0)$.

The pressure observed at point OP is given by the expression

$$P(r, \theta) = \sum_n \int_{\frac{-a}{2}}^{\frac{a}{2}} w(n) \frac{P_0}{4\pi r\prime} e^{j(\omega t - kr\prime - 2n\phi)} dx \tag{A.1}$$

where $r\prime$ is the distance from the position x on an element to the observation point, and P_0 is the pressure amplitude of the point source, related to the initial velocity.

$$r\prime = r - x \sin\theta - nd \sin\theta \tag{A.2}$$

where r is the distance of to the observation point from the center of the array.

In Equation (A.1), we replace $2n\phi$ with $nk_0 d \sin\theta_0$. In the denominator of the integrand, we replace $r\prime$ with r. In the exponent, we replace $r\prime$ with the full expression given in Equation (A.2):

$$P(r,\theta) = \sum_n \int_{\frac{-a}{2}}^{\frac{a}{2}} w(n) \frac{P_0}{4\pi r} e^{j(\omega t - kr + kx\sin\theta + knd\sin\theta - k_0 nd\sin\theta_0)} dx \tag{A.3}$$

Then we move terms out of the sum or integral as we can:

$$P(r,\theta) = \frac{P_0 e^{j(\omega t - kr)}}{4\pi r} \underbrace{\sum_n w(n) e^{jnd(k\sin\theta - k_0\sin\theta_0)}}_{\text{DTFT of weights},w(n)} \int_{\frac{-a}{2}}^{\frac{a}{2}} e^{jkx\sin\theta} dx \tag{A.4}$$

The sum and integral terms can be simplified separately. The sum term is a discrete time Fourier transform (DTFT) of the element weights, where the general DTFT is expressed as $X_d(\lambda) = \sum_{n=-\infty}^{\infty} x_n e^{-j\lambda n}$. In our expression, $\lambda = -d(k\sin\theta - k_0\sin\theta_0)$. Evaluating the integral results in a sinc function.

In order to continue the derivation of equations in Chapter 2, we choose a rectangular weighting function so that $w(n) = 1$ for all n. We could choose a different weighting function, such as a Hanning window, that would produce lower sidelobes:

$$P(r,\theta) = \frac{P_0 e^{j(\omega t - kr)}}{4\pi r} \frac{\sin(\frac{Nd}{2}(k\sin\theta - k_0\sin\theta_0))}{\sin(\frac{d}{2}(k\sin\theta - k_0\sin\theta_0))} \frac{\sin(\frac{ka}{2}\sin\theta)}{\frac{ka}{2}\sin\theta} \tag{A.5}$$

If a is very small, i.e., if the elements of the array are point elements, the final sinc function is equal to 1 for all angles, corresponding to Equation (2.9) of Chapter 2.

Now we assume we have an array of point sources, we remove the assumption that the weighting function is rectangular, and we evaluate the sum term of Equation (A.4) to find weights for an amplitude-steered array with lowered sidelobes. Note that common windows used to lower sidelobes are symmetric, so that $w(n) = w(-n)$. We begin with the expression

$$H(\theta) = \frac{1}{N} \sum_n w(n) e^{jnd(k\sin\theta - k_0\sin\theta_0)} \tag{A.6}$$

where N is assumed to be odd and n takes values from $\frac{-(N-1)}{2}$ to $\frac{N-1}{2}$. We replace $\frac{kd}{2}\sin\theta$ with u, and $\frac{k_0 d}{2}\sin\theta_0$ with ϕ. Combining terms with the same absolute value of n gives

$$H(\theta) = \frac{1}{N} \left\{ w(0) + \sum_{n=1}^{\frac{N-1}{2}} 2 * w(n) \cos(n(u - \phi)) \right\} \tag{A.7}$$

Using the trigonometric identity for the cosine of a sum, we obtain

$$H(\theta) = \frac{1}{N}\left\{ w(0) + \sum_{n=1}^{\frac{N-1}{2}} 2 * w(n)\cos(nu)\cos(n\phi) + 2 * w(n)\sin(n\phi)\sin(nu) \right\} \quad \text{(A.8)}$$

The terms in the sum can be written to give the same form as Equation (2.5), but we can also see from this form the weights for the combination of beamsteering and sidelobe reduction. Where in Chapter 2 we used weights of $\cos(2n\phi)$ and $\sin(2n\phi)$ for arrays with an odd number of elements, we now have weights of $w(n)\cos(2n\phi)$ and $w(n)\sin(2n\phi)$. Using this derivation, we have shown how to calculate amplitude weights for the combination of beamsteering and sidelobe reduction.

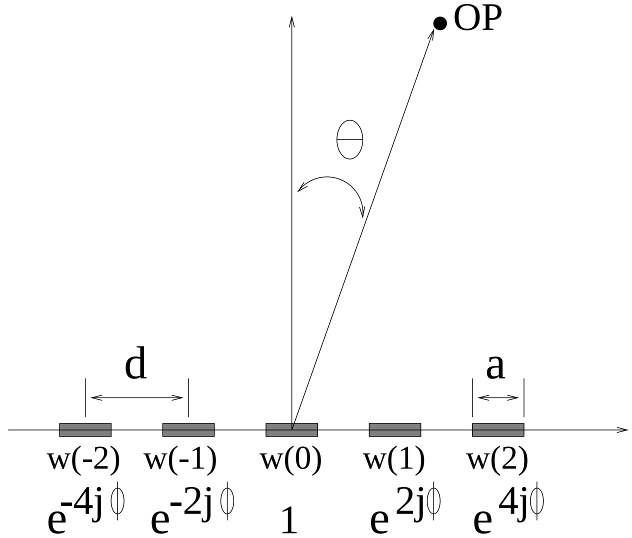

Figure A.1: Definition of variables for derivation.

APPENDIX B

SIMULATION CODE

The code for simulating data collection is presented in this appendix. This program simulates transmitting on cosine elements and receiving on cosine elements. Similar code is used to simulate the other three cases.