CHAPTER 1

INTRODUCTION TO THE NONLINEAR INDEX PROBLEM

Over the past few decades, the propagation of acoustic signals through the human body has found application in medical imaging as well as the treatment of various ailments. Also, many new applications are being continually developed. As the demand for medical ultrasonic devices increases, so does the demand for reliable methods of characterizing the pressure output of the ultrasound transducers. Currently, every focal region is characterized at every transducer voltage setting by directly measuring the acoustic pressure in a water bath. This process is highly time consuming, ultimately increasing the cost and development time for each transducer.

In order to reduce this measurement time, there is a desire to linearly extrapolate the focal pressures from the applied voltage settings. Ideally, the manufacturer would measure the pressure for one voltage setting and then extrapolate to find the pressure for all other voltage settings. Unfortunately, sound propagation is a nonlinear phenomenon restricting the range of inputs over which linear extrapolation can be performed. The goal of this thesis was to determine if a reliable nonlinear indicator could be determined to measure the amount of nonlinearity in the acoustic waveform at the focus. The diagnostic ultrasound equipment manufacturer would then know over which applied voltages linear extrapolation would be valid.

1.1 Background: Extrapolation in Ideal Linear Case

Before addressing the nonlinear problems associated with linear extrapolation, it is important to understand how the extrapolation would work in the ideal case. This will be done by briefly explaining how an applied voltage to the transducer relates to the resulting pressure at the focus of an ideal circularly focused transducer assuming ideal linear propagation. Therefore it is necessary to explain the pressure versus voltage characteristics of a piezoelectric transducer as well as the propagation characteristics of a focused source. Our analysis will be restricted to a focused spherical transducer, but the same ideas also hold for the arrays commonly encountered in medical imaging applications.

The pressure versus voltage characteristics of any piezoelectric transducer can be understood based on the circuit model developed by Krimholtz, Leedom, and Matthaei in 1970 known as the KLM model [*Krimholtz et al.*, 1970]. A diagram illustrating this model is shown in Figure 1.1.

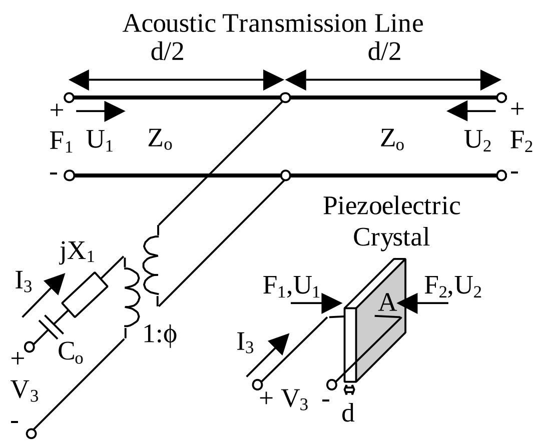

Figure 1.1: The KLM model of a piezoelectric transducer.

In this model, V_3 and I_3 are the respective voltage and current applied to the piezoelectric crystal which produce the resulting acoustic forces, F, and particle velocities, U, at the respective faces of the crystal. The model parameters include the thickness of the crystal, d, the area of the crystal, A, and the characteristic impedance of the acoustic transmission line (i.e. the radiation impedance) modeling the piezoelectric crystal, Z_o. A detailed discussion of acoustical impedances can be found in [*Kinsler et al.*, 2000]. In order to complete the model, it is also necessary to include a capacitor, C_o, an impedance, jX_1, and a transformer with the ratio (1: *f*) that converts the electrical signal into the appropriate acoustical values. C_o results from the resonator consisting of a dielectric, the piezoelectric crystal, between two excited conducting surfaces. The values for these parameters as given by *Krimholtz et al.* [1970] are

$$
\begin{aligned}
Z_o &= rcA \\
C_o &= \frac{eA}{d} \\
X_1 &= \frac{h^2}{w^2 Z_o}\sin\left(\frac{w \cdot d}{c}\right) \\
f &= \frac{wZ_o}{2h}\operatorname{cosec}\left(\frac{w \cdot d}{2c}\right)
\end{aligned}
\tag{1.1}
$$

where e is the permittivity of the piezoelectric under no applied voltage, h is the piezoelectric pressure constant for the crystal, r is the density, and c is the speed of longitudinal sound waves in the crystal.

Now that the model is in place, we can proceed with the analysis of the pressure versus voltage characteristics of the typical transducer. Let some voltage waveform be applied to the terminal V_3. Each frequency component in the waveform will "see" a different impedance set by the values of C_o, X_1, and the impedance "seen" looking into the acoustic transmission line reflected across the transformer. The effective impedance of the acoustical transmission line will depend on the acoustical impedance of the material into which the piezoelectric radiates as well as the frequency. Based on these impedances, a current I_3 will be set up in the transformer. This current will scale linearly with the applied voltage provided that the frequency spectrum also scales linearly as the voltage is changed.

The current I_3 will then be translated to the other side of the transformer, where it will establish the appropriate particle velocities in the acoustic transmission line. The particle velocity waveforms generated at each surface of the crystal can then be determined by summing the currents at the central node and applying basic transmission line theory; both of which are linear operations. The pressure wave can be determined from the velocity wave by simply multiplying by the radiation impedance at the surface of the piezoelectric crystal [*Kinsler et al.*, 2000]. More details on this calculation are provided in Appendix B. Obviously, the resultant acoustical signal will scale linearly with the applied voltage provided that the frequency spectrum of the applied voltage waveform scales linearly with the voltage amplitude.

At this point, it is clear how an applied voltage will linearly translate to an acoustic waveform at the surfaces of the piezoelectric crystal. We can now proceed with the analysis in terms of acoustical signals to see how these waveforms relate to the pressure at the focus of the transducer under ideal linear propagation. The basic layout for a focused transducer is shown in Figure 1.2.

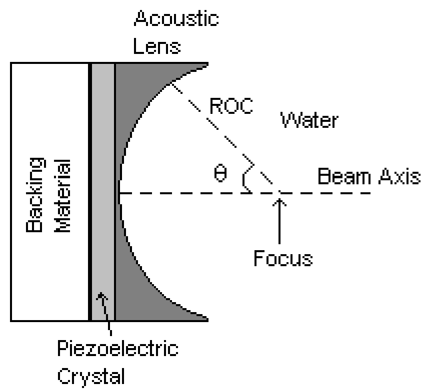

Figure 1.2: Diagram of a typical focused transducer.

The pressure wave propagating out of the piezoelectric crystal and into the acoustic lens is directly related to the applied voltage waveform as was discussed above using the KLM model. The generated acoustic signal will propagate through the acoustic lens until it reaches the lens water interface. At this interface, the incident particle velocity will generate a reflected wave, which will not be analyzed in our discussion, and a transmitted wave. Both waves will be directly proportional to the wave leaving the piezoelectric crystal with the constants of proportionality being the reflection and the transmission coefficients corresponding to the lens water interface. These coefficients would vary across the surface of the acoustic lens since the incident angle q of the plane wave with respect to the surface normal would be different for each point along the surface. This would result in a decrease in the wave amplitude as the angular distance from the lens center increased. However, for each point on the lens, the idea of proportionality still holds.

After leaving the lens, the transmitted wave will be a converging wave, which will focus at a distance, F, given by [*Kinsler*, 2000; *Hecht*, 1998]

$$F = \frac{\frac{1}{c_{water}}}{\frac{1}{c_{water}} - \frac{1}{c_{lens}}} \cdot ROC \tag{1.2}$$

In this equation, c_{water} and c_{lens} are the corresponding longitudinal wave speeds of the sound in the water and in the lens material, and ROC is the radius of curvature for the lens. However, the speed of sound in the lens is typically much greater than that for water, so the above expression if often approximated as

$$F \cong ROC \tag{1.3}$$

This expression will be used in the remainder of the analysis.

Using the approximation in Equation (1.3) the waveform at the focus can be calculated from the transmitted waveform from Poisson's theorem [*Pierce*, 1991]. A complete statement of the theorem is provided and proved in Appendix A. The theorem states that the pressure at the center of a spherical region can be determined from the pressure waveform at all points on the boundary of the region using

$$p(\vec{x}_o, t_o) = \left[\left(\frac{\partial}{\partial R} + \frac{1}{c}\frac{\partial}{\partial t} \right) R \cdot \overline{p}(\vec{x}_o, R, t) \right]_{t = t_o - R/c} \tag{1.4}$$

where R is the radius of the spherical region centered at $\vec{x}_o$, and $\overline{p}(\vec{x}_o, R, t)$ is the spherical mean of the pressure, $p(\vec{x}, t)$, over the spherical boundary given by

$$\overline{p}(\vec{x}_o, R, t) = \frac{1}{4p \cdot R^2} \iint p(\vec{x}_o + \vec{n}R, t)\, dS \tag{1.5}$$

where $\vec{n}$ is the outward unit normal vector of the surface of the spherical region. Equations (1.4) and (1.5) can be used directly to find the pressure waveform at the focus of the transducer by setting $R = ROC$. Notice that linear changes in the pressure at the surface of the transducer, $p(\vec{x}_o + \vec{n}ROC, t)$, translate to linear changes in the spherical mean of the pressure, $\overline{p}(\vec{x}_o, ROC, t)$. Furthermore, linear changes in $\overline{p}(\vec{x}_o, ROC, t)$ result in linear changes in the pressure at the focus, $p(\vec{x}_o, t_o)$, since Equation (1.4) is a linear partial differential equation (PDE).

From this analysis, it is clear that the pressure at the focus of a focused transducer should scale linearly with the applied voltage in the ideal case. Hence, linear extrapolation to determine pressure values at the focus would also work. Unfortunately, the real world is not ideal, and nonlinearities soon distort the simple linear relationship described above. A brief analysis of these nonlinearities is provided in the next section.

1.2 Motivation: Problems Introduced by Acoustical Nonlinear Effects

As was discussed in Section 1.1, it should be possible to predict the acoustic fields at the focus of a focused transducer from values at other voltage settings in the ideal linear case. One possible nonlinear mechanism, source nonlinearities, was also mentioned. Obviously, if the spectrum of the applied voltage pulse changes with voltage amplitude, the resulting pressure at the focus will also not scale with applied voltage. However, the propagation of the acoustic signal will also introduce nonlinearities, even when a perfect linear voltage source is used in the experiment. In this section, the problems introduced by these acoustical nonlinearities will be briefly explained. The acoustical nonlinearities served as a motivation for this thesis since the goal was to find a reliable indicator for when they could be neglected and linear extrapolation could be performed.

One nonlinear effect that limits our ability to perform voltage-based linear extrapolation is asymmetric distortion. When diffraction and nonlinearity both act on an acoustic wave, the peak compressional pressure is always larger than the peak rarefractional pressure as shown in Figure 1.3.

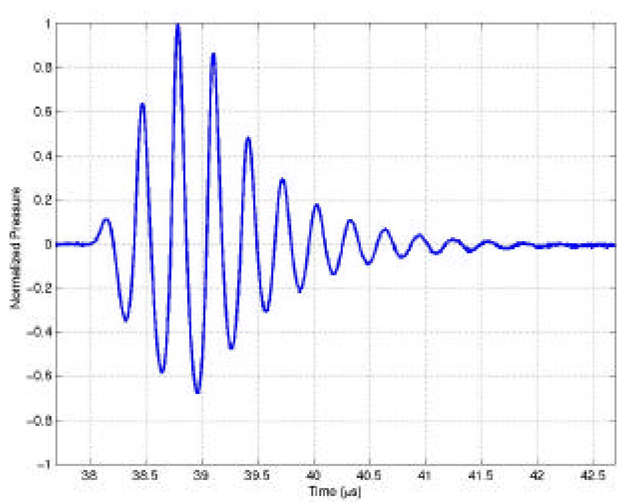

Figure 1.3: Plot illustrating asymmetric distortion for a focused source.

This type of pulse asymmetry is known as *asymmetric distortion*. A complete discussion of this effect is provided in Chapter 2. For now, however, it is sufficient to know that the peak compressional pressures will be larger and the peak rarefractional pressures will be smaller than those predicted by linear extrapolation.

Another nonlinear effect that corrupts linear extrapolation is nonlinear absorption, which will be discussed in greater detail in Chapter 3. Nonlinear absorption is commonly analyzed in terms of acoustical saturation. Saturation is the process by which the amplitude of a propagating acoustic wave becomes independent of its initial amplitude at the source due to the nonlinear generation and absorption of harmonics [*Hamilton and Blackstock*, 1998]. In general, nonlinear absorption will lower the peak compressional pressures, the peak rarefractional pressures, and the total peak-peak pressures from those predicted by linear extrapolation. Notice that nonlinear absorption and asymmetric pulse distortion are competing effects whose relative influence can currently only be assessed experimentally.

1.3 Approach and Summary of Results

In order to determine when acoustical nonlinearities could be neglected for the purpose of linear extrapolation, many different indicators of nonlinearity have been proposed. The goal of this work was to assess the proposed indices to determine if one of them could then be used as a guide for the linear extrapolation. To this end, the theoretical basis for each of the indices was investigated. However, all the of indices were found to be based on approximations of the complete nonlinear acoustic equations. Unfortunately, this is unavoidable due to the complexity of the nonlinear partial differential equations involved. Therefore, the different indices could only be assessed experimentally.

Before the indices could be assessed, however, a reliable extrapolation factor needed to be found. This was done by performing a series of experiments with a hydrophone placed very close to a series of transducers. Several different extrapolation factors were then compared in terms of their ability to perform linear extrapolation for these experiments. Errors in these measurements could also be used as a confidence measure for the evaluation of the nonlinear indices.

After finding a viable extrapolation factor, a series of experiments were performed where the hydrophone was placed at the focus of a transducer. Threshold values for the different indicators corresponding to the maximum allowable amount of nonlinearity in the waveform were then found for each transducer and drive condition. These threshold values were then evaluated in terms of their consistency between transducers and drive conditions as well as their behavior within each data set.

The results of our investigation showed that none of the proposed indices would be able to govern linear extrapolation for all pressure measures of the focal waveform. This occurred because they are based on theories that either ignore nonlinear absorption or ignore asymmetric distortion. There is no theory that attempts to capture both effects. Therefore, there is a definite potential for future development in this area. Furthermore, if a more complete theory were to be developed, it might also be able to generate a usable indicator of nonlinearity to govern the extrapolation procedure.