

APPENDIX D: COMPARISON OF THE RLC CIRCUIT MODEL TO THE KLM MODEL

In this appendix, the frequency characteristics of the RLC circuit model and the KLM model are compared via numerical simulations. To facilitate the comparison, the piezoelectric crystal was assumed to be lossless, air backed, and radiating into a homogeneous lossless region. This was done to simplify the analysis since closed form expressions for the element values in the RLC circuit model have already been derived for this configuration by *O'Brien* [2000] and are provided in Appendix C. The crystal used in the simulations had a crystal resonance, $\omega_o = \frac{cP}{d}$, of 5.5 MHz, and the piezoelectric was taken to be lithium niobate with the appropriate property values provided by (http://valpeyfisher-ud.com/content/tech.lithium_niobate.html). The simulated crystal was similar to the 5.5 MHz transducers evaluated in the experiment. Three different loading conditions were simulated by changing the characteristic impedance of the homogeneous region into which the crystal radiated.

Comparison of Input Impedance

In this section, the input impedances for a piezoelectric crystal under the three different loading conditions are found based on the KLM model and the RLC circuit model. Since the admittance better illustrates the resonant behavior of the transducer, the results are compared in terms of the real and imaginary parts of the input admittance, Y_{in} . The plots illustrating all three loading conditions are provided in Figure D.1. Notice that when the loading is small, characteristic impedance of 2.5 Mrayl, the impedance curves for both models are almost identical. However, as the loading increases, the models are only the same for a smaller range of frequencies about the resonant frequency. Also, whereas the KLM model clearly exhibits a zero and resonance at the second and third harmonic frequencies, respectively, both features are absent from the RLC circuit model.

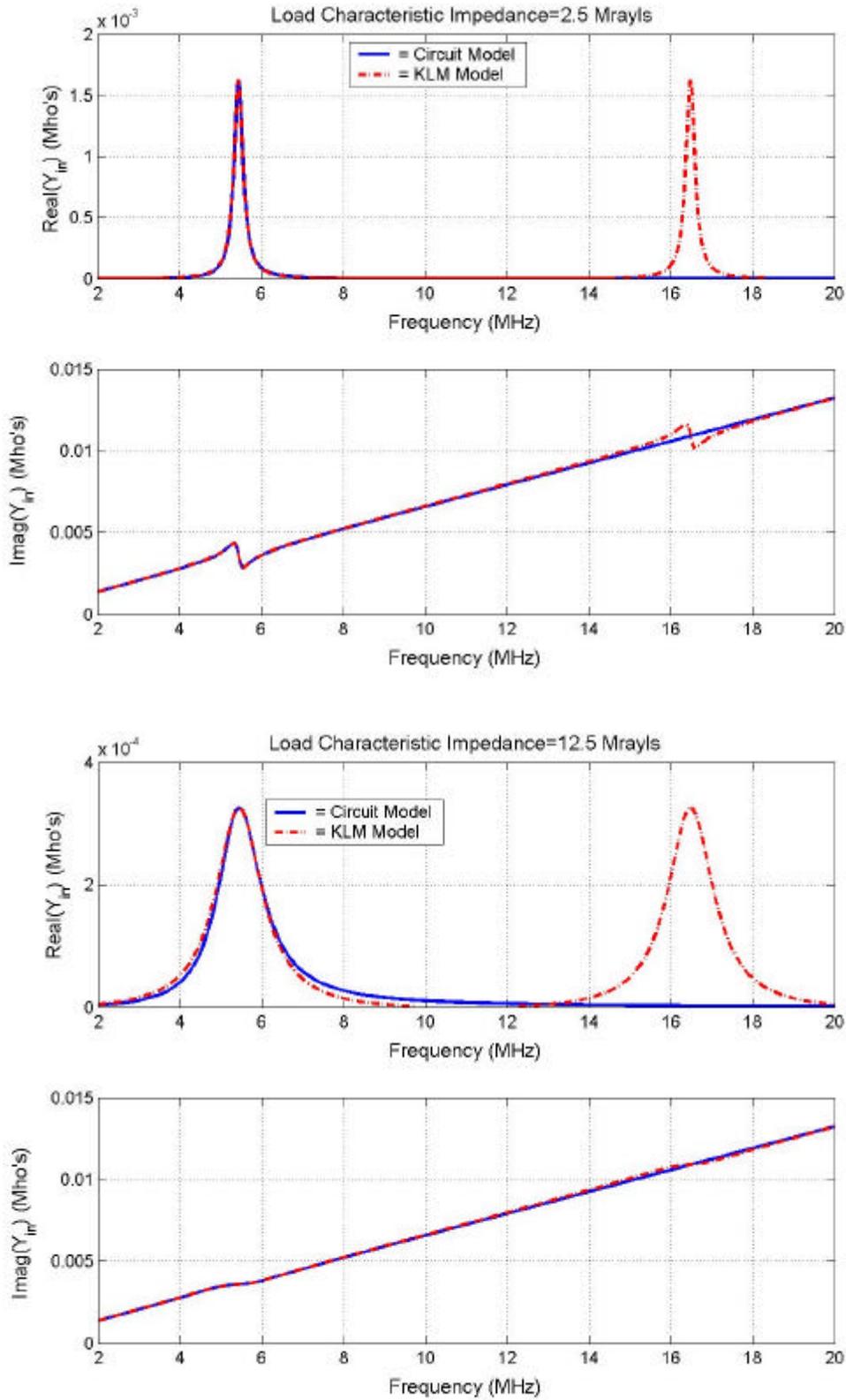


Figure D.1: Input admittance for the KLM model and the RLC circuit model.

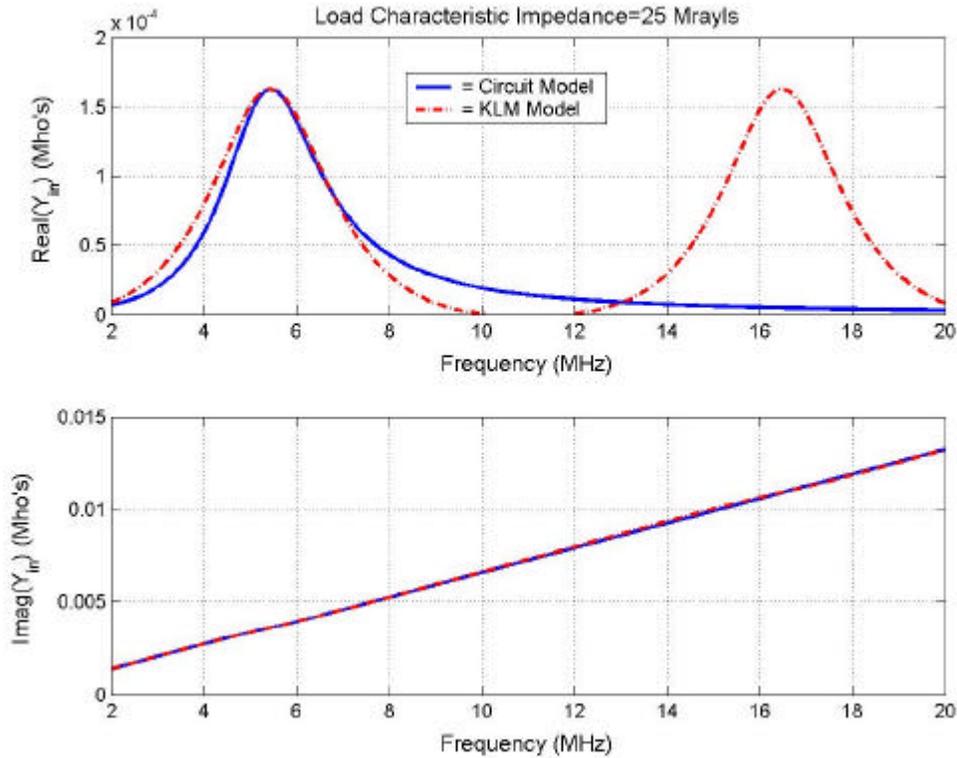


Figure D.1: (Continued).

Comparison of Transfer Functions

In this section, the transfer function for a piezoelectric crystal under the three different loading conditions are found based on the KLM model and the RLC circuit model. For the KLM model, the output is the radiated acoustic signal, and for the circuit model, the output is the voltage across the resistor, R_l . The resulting transfer characteristics are shown in Figure D.2. Notice once again that there is good agreement in both the magnitude and the phase for small loading, but the agreement degrades as the loading increases. Also, notice that the peak output is located at a frequency of approximately 5.44 MHz, which is different than then the crystal resonant frequency of 5.5 MHz. Another interesting feature is that for large loading, characteristic impedance of 25 Mrayl, the peak output frequency for the KLM model at about 5.43 MHz, is less than the peak output frequency of the RLC circuit model of about 5.44 MHz. This same downshift also occurs in the admittance plots provided in the previous section. Since the values of the RLC circuit parameters are derived from the physics of the crystal for this

configuration *O'Brien* [2000], the downshift that is predicted by the KLM model is probably not physical. However, a complete analysis of the models and this effect is beyond the scope of this thesis.

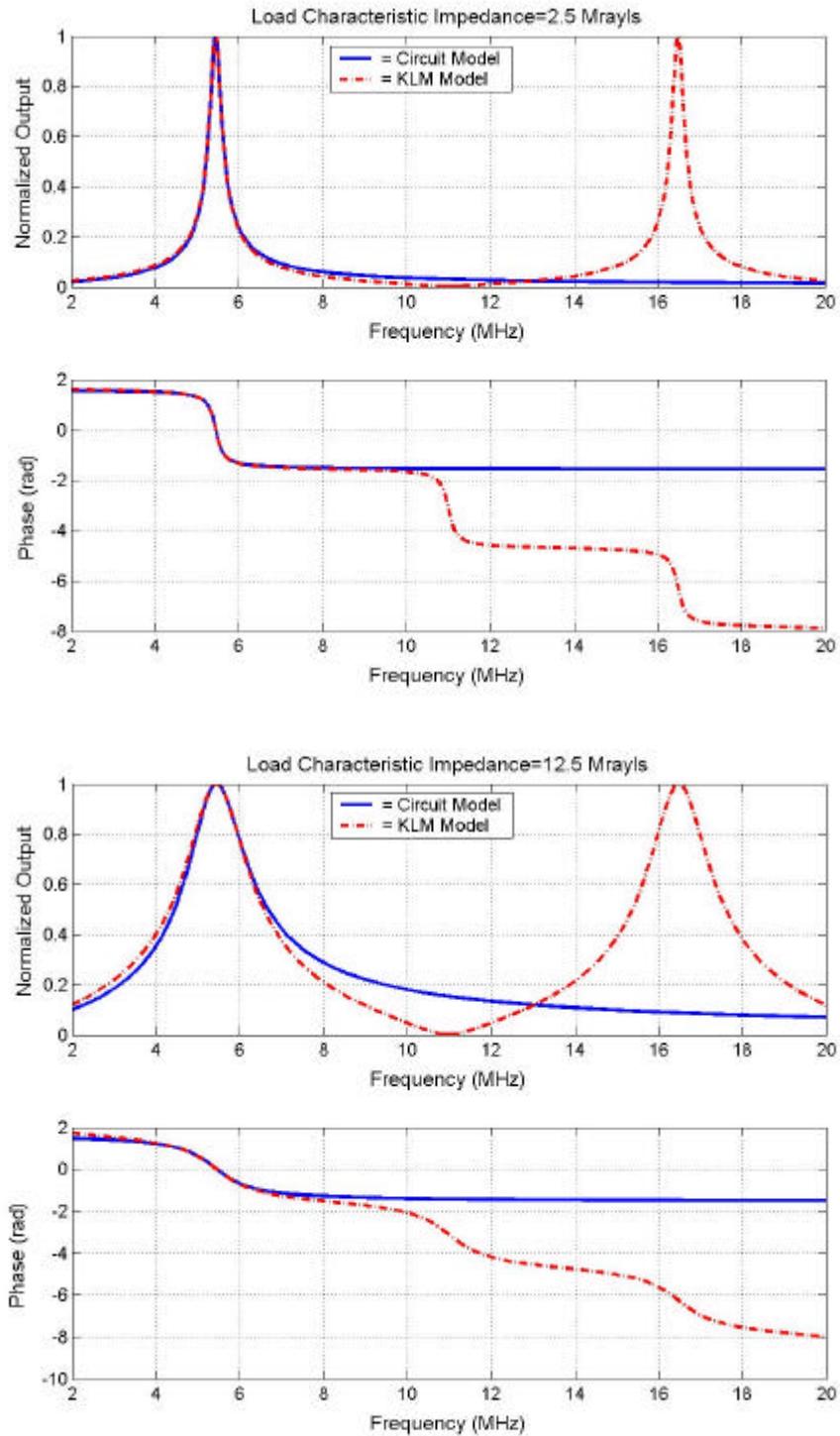


Figure D.2: Transfer function for the KLM model and the RLC circuit model.

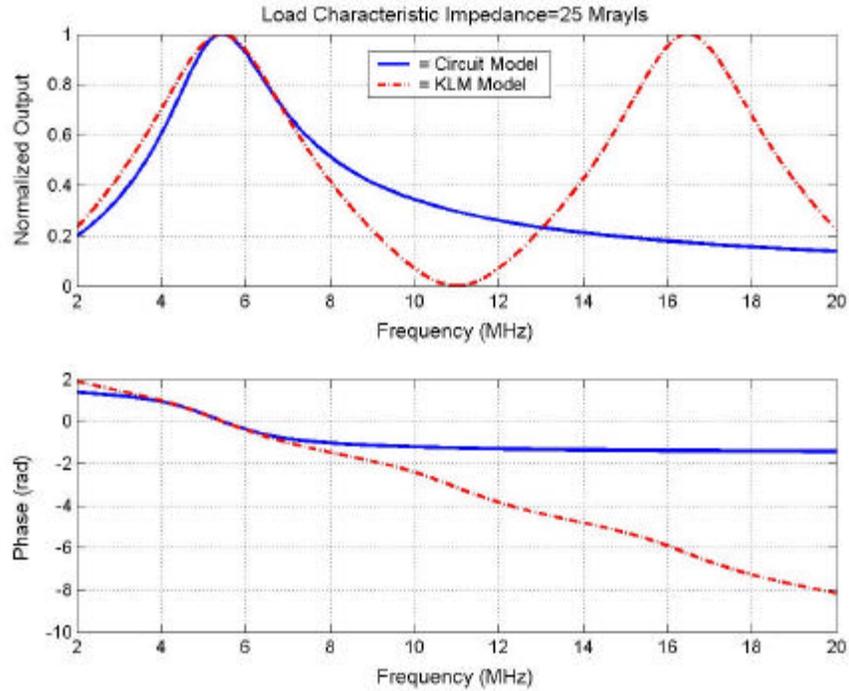


Figure D.2: (Continued).

MATLAB Code Used to Perform Simulations:

Main Program

%The goal of this program is to simulate and compare the RLC
 %circuit model to the KLM model in terms of input impedance
 %and transfer functions.

```
clear all
close all
j=sqrt(-1);
```

```
%Set Assumed Transducer Parameters
c=7316; %Speed of sound in crystal (m/s)
rho=4640; %Crystal Density (kg/m^3)
```

```
fo=5.5e6; %Crystal Resonance Frequency (Hz)
R=0.95e-2; %Transducer radius in m
```

```
d=c/(2*fo); %Cyrstal thickness (m)
A=pi*(R)^2; %Surface area of crystal (m^2)
```

```
epsilon=2.4703e-10; %Permittivity (F/m)
```

```

Zo=rho*c*A;

h=5.1e9; %Piezoelectric Pressure Constant (N/C)

%Set Load Impedances
c1=0; %Speed of sound in material to the left (m/s)
rho1=0; %Density of material to the left (kg/m^3)

Z1=c1*rho1*A;

c2=500; %Speed of sound in material to the left (m/s)
rho2=5000; %Density of material to the left (kg/m^3)

c2*rho2/1e6 %Show current characteristic impedance of load.

Z2=rho2*c2*A;

label='Load Characteristic Impedance=2.5 Mrayls';

%*****
*
%Determine what the circuit values should be based on the
%paramters used in the model.
%(Assumes Piezoelectric is Air Backed)

Co=epsilon*A/d;

C1=8*d*(Co*h)^2/(A*rho*(pi*c)^2-8*Co*d*h^2);

L1=0.5*rho*d*A/((2*Co*h)^2);

R1=Z2/((2*Co*h)^2);

%*****
*
%Perform Simulations
f=(2:0.01:20)*1e6;

for fli=1:length(f)
    %Find for KLM model
    Zout_KLM(fli)=find_Zout(f(fli),Zo,Co,h,d,c,0,Z2);

    %Find for ckt model
    w=2*pi*f(fli);

    Zp1=R1 + j*w*L1 + 1/(j*w*C1);

```

```

Zout_ckt(fli)=(j*w*Co + 1/Zp1)^-1;

%Find output Voltage for each frequency w/ ckt model.
Vout(fli)=(R1/Zp1);

%Find the output particle velocity based on the KLM
model
[Ua1,Ua2(fli)]=find_TR_out(f(fli),1,Zo,Co,h,d,c,0,Z2);

end

figure(1)
clf
subplot(211)
plot(f*1e-6,real(1./Zout_ckt))
hold
grid
plot(f*1e-6,real(1./Zout_KLM),'r-.')
xlabel('Frequency (MHz)')
ylabel('Real(1/Z_o_u_t)')
legend('= Circuit Model','= KLM Model')
title(label)

subplot(212)
plot(f*1e-6,imag(1./Zout_ckt))
hold
grid
plot(f*1e-6,imag(1./Zout_KLM),'r-.')
xlabel('Frequency (MHz)')
ylabel('Imag(1/Z_o_u_t)')
legend('= Circuit Model','= KLM Model')

figure(2)
clf
subplot(211)
plot(f*1e-6,abs(Vout)/max(abs(Vout)))
hold
grid
plot(f*1e-6,abs(Ua2)/max(abs(Ua2)),'r-.')
xlabel('Frequency (MHz)')
ylabel('Normalized Output')
legend('= Circuit Model','= KLM Model')
title(label)

subplot(212)
plot(f*1e-6,phase(Vout))

```

```

hold
grid
plot(f*1e-6,phase(Ua2),'r-.')
xlabel('Frequency (MHz)')
ylabel('Phase (radians)')
legend('= Circuit Model','= KLM Model')
title(label)

```

KLM Impedance Function

```
function [Zout]=find_Zout(freq,Zo,Co,h,d,c,Z1,Z2)
```

```
%This is a MATLAB function that finds the impedance seen
%looking into the terminals of a transducer at a specific
%frequency. The calculations are based on the KLM model.
```

```
%Inputs
```

```
% freq = frequency driving transducer (Hz)
% Zo = impedance of Acoustic TR line
% Co = static capacitance of transducer (F)
% h = piezoelectric constant
% d = crystal thickness (m)
% c = speed of sound in crystal (m/s)
% Z1, Z2 = load impedances.
```

```
%Output
```

```
% Zout = the impedance seen looking into the terminals
of % the simulated transducer.
```

```
%*****
```

```
*
```

```
w=2*pi*freq;
j=sqrt(-1);
```

```
%Find X1
```

```
X1=(h^2/(Zo*w^2))*sin(w*d/c);
```

```
%Find phi
```

```
phi = (w*Zo/(2*h))*csc(w*d/(2*c));
```

```
%Find Za
```

```
Zin1=Zo*(Z1+j*Zo*tan(w*d/(2*c)))/(Zo+j*Z1*tan(w*d/(2*c)));
Zin2=Zo*(Z2+j*Zo*tan(w*d/(2*c)))/(Zo+j*Z2*tan(w*d/(2*c)));
```

```
Zin=1/(1/Zin1 + 1/Zin2);
```

```
Za=Zin/(phi^2);
```

```
Zout=(j*X1 + Za + 1/(j*w*Co));
```

KLM Radiation Function

```
function [U1,U2]=find_TR_out(freq,V3,Zo,Co,h,d,c,Z1,Z2)
```

```
%This is a MATLAB function that finds the particle  
%velocities of the acoustic waves flowing out from the  
%surfaces of a piezoelectric source. The calculations are  
%based on the KLM model.
```

```
%Inputs
```

```
% freq = frequency driving transducer (Hz)  
% V3 = voltage across the transducer (V)  
% Zo = impedance of Acoustic TR line  
% Co = static capacitance of transducer (F)  
% h = piezoelectric constant  
% d = crystal thickness (m)  
% c = speed of sound in crystal (m/s)  
% Z1, Z2 = load impedances.
```

```
%Output
```

```
% U1 = particle velocity of wave traveling to the left.  
% U2 = particle velocity of wave traveling to the right.
```

```
%%%%%%%%%
```

```
%Find I3%
```

```
%%%%%%%%%
```

```
w=2*pi*freq;
```

```
j=sqrt(-1);
```

```
if w==0
```

```
    I3=0;
```

```
    phi=1000;
```

```
else
```

```
    %Find X1
```

```
    X1=(h^2/(Zo*w^2))*sin(w*d/c);
```

```
    %Find phi
```

```
    phi = (w*Zo/(2*h))*csc(w*d/(2*c));
```

```
    %Find Za
```

```
Zin1=Zo*(Z1+j*Zo*tan(w*d/(2*c)))/(Zo+j*Z1*tan(w*d/(2*c)));
```

```

Zin2=Zo*(Z2+j*Zo*tan(w*d/(2*c)))/(Zo+j*Z2*tan(w*d/(2*c)));

Zin=1/(1/Zin1 + 1/Zin2);

Za=Zin/(phi^2);

Zout=(j*Xl + Za + 1/(j*w*Co));

I3=V3/Zout;
end

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%Find G1 and G2; T1 and T2%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%Find G's: current reflection coefficients
G2=(Zo-Z2)/(Zo+Z2);
G1=(Zo-Z1)/(Zo+Z1);

%Find T's: current transmission coefficients
T1=G1+1;
T2=G2+1;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%Find VFp and VBn%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

K=-(I3/phi)/(exp(j*d*w/c) - G1*G2*exp(-j*d*w/c));

VFp=K*(exp(j*(d/2)*w/c) - G1*exp(-j*(d/2)*w/c));
VBn=K*(G2*exp(-j*(d/2)*w/c) - exp(j*(d/2)*w/c));

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%Find U1 and U2%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

U1=-T1*VBn;
U2=-T2*VFp;

```